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PART I

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FATIGUE STRENGTH DESIGN AND ANALYSIS OF AIRCRAFT STRUCTURES

PART I. SCATTER FACTORS AND DESIGN CHARTS

P. R. ABELKIS

DOUGLAS AIRCRAFT COMPANY, INC.

TECHNICAL REPORT AFFDL-TR-66-197, PART I

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AIR FORCE FLIGHT DYNAMICS LABORATORY
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AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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FOREWORD

This report was prepared by Douglas Aircraft Company, Inc., Aircraft Division, Long Beach, California, under USAF Contract No. AF33(615)-3333. This contract was initiated under Project No. 1367, "Structural Design Criteria", Task No. 136711, "Structural Fatigue Design Criteria". The work was administered under the direction of the Flight Dynamics Laboratory, Research and Technology Division, Mr. D. Simpkins, project engineer.

The Douglas program was conducted under the direction of Mr. M. Stone, Chief Design Engineer of the Structural Mechanics Section, Engineering and Product Development. The work was performed in the Research and Development Methods group by Mr. P. R. Abelkis and Mr. W. P. Bobovski under the supervision of Mr. F. C. Miskam. Mr. P. R. Abelkis was the Douglas project engineer.

This report covers work conducted from December 1965 to September 1966. The manuscript was released by the author in February 1967 for publication as an RTD Technical Report.

This technical report has been reviewed and is approved.

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Chief, Theoretical Mechanics Branch
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ABSTRACT

Aircraft fatigue strength design and analysis concepts were investigated in the areas of fatigue life scatter factors and fatigue strength design-analysis charts.

A fatigue scatter factor is defined as the ratio of the mean life to the life for a specified probability of failure and confidence level. For design purposes, operational life scatter factors are defined in terms of the joint probability distribution of the applied loads spectra variation in a fleet of aircraft and the basic fatigue life scatter represented by fatigue test data. Basic fatigue life scatter properties for aluminum alloy materials and structures were statistically derived from a fatigue test data survey of over 6,000 specimens. The basic scatter derived frequency and probability distributions greatly deviate from the log Normal distribution beyond $\mu \pm 2\sigma$. Several joint probability distribution models illustrate the procedure of calculating operational life scatter factors. An actual aircraft service failure history is accurately predicted by the joint probability distribution concept.

A procedure for the development of fatigue strength design-analysis charts is outlined and illustrated by several examples. The charts, in the form of damage rate curves, are defined by generalized loads spectra parameters and the fatigue quality of the structural element.

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SECTION I

INTRODUCTION

The practical rather than purely statistical and probabalistic aspects of fatigue life scatter of aircraft structures concerns the design engineer and the fatigue analyst. The simple and direct, even though only approximate, fatigue strength check methods interests the design engineer when he is confronted with preliminary design problems, or the fatigue analyst when quick approximate life estimates must be obtained. This report, Part I of two parts of the subject fatigue study of aircraft structures, presents discussions, arguments, recommendations, and supporting data of the fatigue life scatter and general approaches in the development of fatigue strength design charts. Part II of the report presents a complete description of a fatigue life analysis computer program in the form of a user's manual.

Any discussion of analytical fatigue life prediction must firstly note that fatigue life is a random variable and although absolute extremes of performance levels may not be readily resolved, there is a reasonable expectation of assigning some degree of reliability to life prediction. Secondly, the meaning of the term 'fatigue life' must be clearly defined. Fatigue of materials and, in turn, of structures is a form of progressive failure caused by the repeated application of cyclic loads. The failure process can be divided into three basic stages:

1. Sub-microscopic intergranular deformation
2. Appearance of a visible crack
3. Crack propagation

A complete final failure of a structural element can occur during any of these progressive failure stages and it will always be a static failure when an applied load exceeds the design ultimate strength of the element during the first stage, or the residual strength during the second and third stages. This concept is qualitatively illustrated in Figure 1. The structure may represent a single load path element or a complex redundant structure, such as the wing. Regardless of the type of structural element the objective of fatigue strength design criteria should be the design of structures for a specified operational life requirement associated with a realistic minimum probability of fatigue crack initiation. Thus, fatigue strength life defines the time interval during which the probability of initiating a crack is a specified low value. After crack initiation and reduction of the ultimate strength capability of the structural element, the problem becomes a function of the fail-safe design criteria where the probability of the final failure becomes a function of the joint probability of encountering a load which exceeds the residual strength of the structural element. With crack propagation the residual strength decreases and the probability of complete failure increases. The life interval from crack initiation to the time when residual strength reaches the design or 80% limit load level, depending on the fail-safe design criteria, is no more the problem of fatigue strength but of crack propagation rates and redundancy of the structure. Therefore, if the fatigue strength design objectives of any structural element were to design for a safe-life during which the probability of crack initiation was a statistically and realistically acceptable low value, then, also, the probability of complete failure during the required lifetime would be greatly minimized.

The above fatigue strength design criteria concepts are applied to the development of fatigue life scatter factors presented in Section II and the Appendix. Scatter factors, with respect to the mean life, are directly related to probabilities of failure and confidence levels. Section III presents an approach for a possible development of generalized fatigue strength design charts in the form of fatigue damage rate curves as a function of the applied loads spectrum parameters and the fatigue strength quality of the structural element.

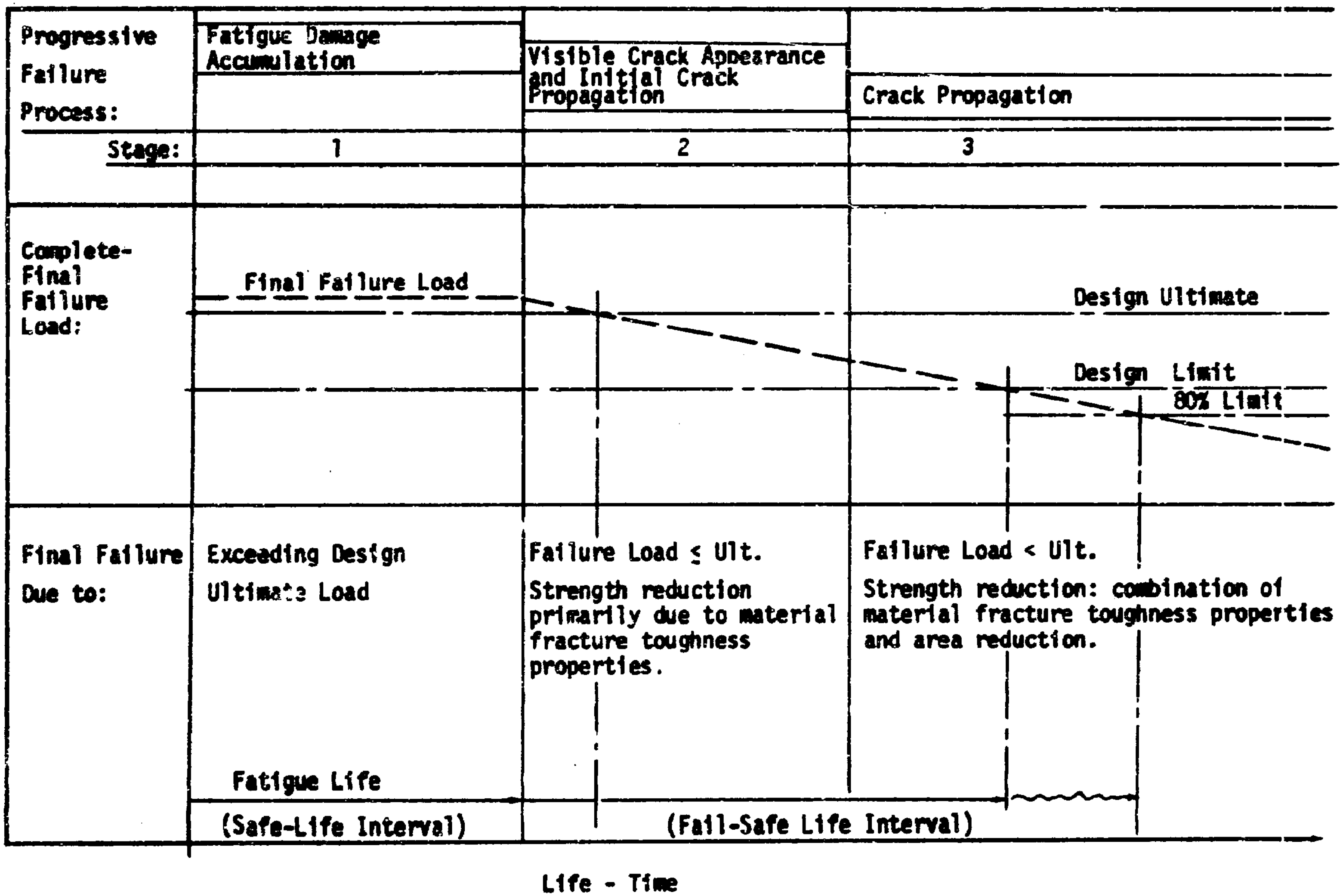


FIGURE 1. PROGRESSIVE FAILURE OF A STRUCTURAL ELEMENT

SECTION II

FATIGUE LIFE SCATTER FACTORS

Fatigue life of aircraft structures is a statistical value and consequently must be evaluated in this context. An estimate of fatigue life must be always associated with a probability and confidence of attaining it, i.e., the reliability at the specified life.

Fatigue life variation of aircraft structures, as represented by a group of aircraft, supposedly identically designed and manufactured to perform a specified envelope of missions, is a function of two principal variables. In general terms, the two variables are:

1. The applied loads and the environment in which the aircraft operate.

2. The structural fatigue strength response under identical loading and environmental conditions.

In the case of fatigue analysis and design of aircraft structures for specified life requirements, the two variables must be considered jointly. It should be noted that in the analytical calculation of fatigue lives, the inaccuracies of analysis methods, or more properly, of the cumulative damage theories used, should not be considered as a contributing factor in the statistical evaluation of the predicted life. The life prediction cumulative damage criteria is a problem in itself and must be treated independently from the statistical evaluation of the actual fatigue life scatter. This study is concerned only with the statistical aspects of fatigue life scatter apart from the inaccuracies of fatigue life prediction methods. The problem delves only with the question of what is the fatigue life scatter magnitude and distribution.

Of the two principal variables contributing to the scatter of fatigue lives, the structural response can be studied independently of the other variable in the form of laboratory fatigue test results. This is true, because test samples can be composed of identical specimens tested under the same loading and environmental conditions. The life scatter exhibited by the laboratory test specimens is to be defined as the "basic fatigue life scatter" and it reflects the effect of material and manufacturing tolerance variables on life scatter.

Life deviation from the mean value is often defined in terms of "scatter factors", "fatigue safety factors", etc., etc. The name is not important. However, the meaning and magnitude of these factors is too often clouded by the divergence of individual interpretations commonly dictated by the objective of attaining a preselected result. Thus, an examination of the actual meaning and application of the fatigue life scatter factors in the fatigue analysis and design of aircraft structures is in order. First, let us define the fatigue life scatter (or safety) factor, in the most general form, as,

$$SF\big|_{p}^{c} = \bar{N}_c/N_p \tag{1}$$

where, $\bar{N}_c$ = Mean Life; subscript c refers to the confidence level.

N_p = Life associated with a probability of failure, p, or a reliability level, R, where R = 1 - p.

Life, N, may represent load cycles, time-flight hours, or any other applicable measure of life. Fatigue life is defined as the time required to initiate a crack which would tend to reduce the ultimate static strength capability of the structural element in its virgin condition. This concept of fatigue life is discussed more fully in Section 1, Introduction. Therefore, design of aircraft structures for specified life requirements implies a design with a minimum probability of crack initiation in the specified lifetime.

There are three basic parameters which must be known in order to define the fatigue life scatter statistical model: mean life, standard deviation, and the frequency or probability distribution. The variable in question, life N, is generally transformed to $\log_{10}N$ in the calculation of these parameters, where, for a given sample of size n, the sample mean and standard deviation are calculated as,

$\overline{\log N_i}$ = Arithmetic mean of log lives

$$= (\Sigma \log N_j)/n, \; j = 1, 2, \ldots n \tag{2}$$

$\bar{N}_i$ = geometric mean life

$$= (N_1 \times N_2 \times \ldots . \times N_n)^{1/n} \tag{3}$$

$$= \text{Antilog } (\overline{\log N_i}) \tag{4}$$

S_i = Standard deviation of log lives

$$= [\Sigma (\log N_j - \overline{\log N_i})^2/(n - 1)]^{1/2} \tag{5}$$

Generally, the Normal-Gaussian frequency distribution with the life log transformation is used to approximate the fatigue life scatter, where the frequency-density distribution is,

$$f(\log N_j) = (1/\sigma\sqrt{2\pi})e^{-[(\log N_j - \overline{\log N})/\sigma]^2/2} \tag{6}$$

where σ and $\overline{\log N}$ are population parameters. However, because of the differences between the Normal and fatigue life scatter distribution in the extreme value ranges, a number of other frequency distributions have been proposed for the statistical analysis of fatigue test data, such as the Weibull distribution function, Reference 1, and the "extreme value" distribution used by Freudenthal and Gumbel, Reference 2. One result of this study is the derivation of an empirical frequency distribution expression for the basic fatigue life scatter of aluminum alloys based on a large collection of fatigue test data, as described in the Appendix.

In all subsequent discussions, reference to the Normal distribution or standard deviation will imply the log Normal distribution and the log standard deviation. Also, $\bar{N}$, as calculated by equation (3) or (4) will be simply referred to as the mean life and, unless otherwise noted, will imply the median life.

1. Fatigue Life Basic Scatter

If a fatigue test is performed on a number of 'identical' specimen, loaded by 'identical' cyclic load time histories in a constant environment, the resulting lives, whether they are defined by the time to crack initiation or final failure, will not be 'identical', they will exhibit a certain amount of scatter. The scatter is due to the fact that neither the specimens nor the loadings are truly 'identical'. Allowing the freedom of saying that the loading is 'identical' for all practical purposes, the scatter becomes a function of the detail diversities of the specimen: variation of the material properties and manufacturing tolerances on the macro and micro levels. The existence of these variations is real and the resulting basic scatter in the fatigue lives of materials and structures is inescapable.

In order to define the typical fatigue life basic scatter of aluminum alloy materials and structures, a survey was made of 1,180 fatigue test samples representing 6,659 specimens. The description of the test data and the results of the survey are presented in the Appendix. The objectives of the test data survey were to check the validity of the Normal frequency distribution as it applies to the basic fatigue life scatter and to define representative standard deviation values for aluminum alloys. The results of the survey were:

1. The Normal distribution is not an accurate representation of the fatigue life basic scatter, in particular for lives beyond $\pm 2\sigma$ from the mean, see Figures 21 to 31 in the Appendix, where σ is the population standard deviation. On the basis of the test data surveyed, the following expressions were derived as representative of fatigue life basic scatter,

Frequency Distribution:

$$f(x) = C_1 e^{-d_1 |x|} + C_2 e^{-d_2 |x|} + C_3 e^{-d_3 |x|} \qquad (7)$$

where,

$$x = (\log N - \overline{\log N})/\sigma$$

$$\sigma = [\Sigma(\log N - \overline{\log N})^2/(n-1)]^{1/2}$$

and,

$$f(-x) = f(x)$$

Cumulative Probability Distribution:

$$F(-x) = A_1 e^{-d_1 |x|} + A_2 e^{-d_2 |x|} + A_3 e^{-d_3 |x|}, \quad x \leq 0 \tag{8}$$

and,

$$F(x) = 1 - F(-x), \quad x > 0$$

where A, C, and d are constants, a function of σ, with a recommended upper limit of $\sigma = 0.75$:

$$A_1 = 1.587\sqrt{\sigma} \qquad d_1 = 1.3 + 0.86\sqrt{\sigma}$$

$$A_2 = 0.015 \qquad d_2 = 0.28 + 0.44\sqrt{\sigma}$$

$$A_3 = 0.485 - 1.687\sqrt{\sigma} \qquad d_3 = 1.09 + 2.16\sqrt{\sigma}$$

and,

$$C_1 = A_1 d_1,\ C_2 = A_2 d_2,\ C_3 = A_3 d_3$$

The differences between the Normal and the derived distributions are clearly illustrated by Figures 32 and 33 in the Appendix. Table 1 presents probability of failure values of the derived distribution for selected σ values. If it is assumed that the basic fatigue scatter has a universal distribution, then, equations (7) and (8), based on aluminum alloys test data, can be also considered to be applicable to other materials.

2. Under constant amplitude loading the standard deviation varies as a function of life and specimen type, see Figure 38 in the Appendix. These standard deviation values are recommended for use as representative population standard deviations in the statistical evaluation of fatigue test S-N data.

3. Under spectrum loading, a population standard deviation of 0.14 is recommended for use in the statistical evaluation of the basic life scatter of notched specimen and structures.

If, for the moment, the population true mean life, $\bar{N}$, and the standard deviation, σ, are assumed to be known, basic fatigue life scatter factors with respect to the mean life, for a specified probability of failure, p, can be calculated as,

$$SF|_p = \bar{N}/N_p \tag{9}$$

N_p = Life corresponding to a specified probability of failure, where the relationship between N and N_p is,

$$\log N_p = \overline{\log N} - m_p\sigma,\ N_p < \overline{N} \quad (10)$$

m_p - Number of standard deviations from the mean for a specified cumulative probability of failure.

and the scatter factor can be calculated as a function of m_p and σ,

$$\log N_p - \overline{\log N} = - m_p\sigma$$

$$\overline{\log N} - \log N_p = m_p\sigma$$

$$\log (\overline{N}/N_p) = m_p\sigma$$

$$SF|_p = (\overline{N}/N_p) = \text{Antilog } (m_p\sigma),\ N_p < \overline{N} \quad (11)$$

$$= (N_p/\overline{N}) = \text{Antilog } (m_p\sigma),\ N_p > \overline{N}$$

Figure 2 presents the basic fatigue life scatter factors with respect to the mean life, as calculated by equation (11), for the Normal and the test data, equation (8) probability distributions for selected values of σ. The relatively large differences between the Normal and test data distribution scatter factors as well as the high scatter factors of the test data distribution at low probabilities of failure must be viewed in the light of relatively large samples of data, in effect, theoretically, of sample sizes approaching infinity.

1.1 Mean Life Estimation. If a number of tests are performed on 'identical' specimen under 'identical' loadings, the resulting test data sample of size n provides information for the estimation of the intervals or regions which, with a certain confidence level, can be expected to contain the true population parameters of interest: mean life, $\overline{N}$, and standard deviation, σ. The interval decreases with increase in sample size and decrease in confidence level. As pointed out in References 1 and 6, for any reasonable estimate of the population parameters, sample sizes of at least n = 3 or 4 and n = 10 are needed for the estimation of $\overline{N}$ and σ, respectively. The concept of a confidence interval is often stated as: "For a given confidence level, c, the probability that the true population parameter lies within the interval so calculated, is c." In other words, if the intervals with confidence level, c, were calculated for a large number of samples which came from the same population, the true population parameter would be included in 'c' per cent of these intervals.

For the population which is normally distributed, the population mean confidence interval or region can be calculated from the sample data in a number of different ways which, unfortunately, give the same number of different results. Before listing several of these expressions, it should be noted that in the fatigue life calculations and predictions the main interest lies in the lives shorter than the mean and associated low probabilities of failure. Consequently, there is no reason to consider the confidence on an interval, but rather, the confidence on the minimum value of the interval. In the following discussion, the notation for confidence, c, will imply the singular confidence limit, where the relationship between the confidence interval level, γ, and c, is

$$c = (1 + \gamma)/2 \tag{12}$$

where, c and γ are proportions, $0 < (c, \gamma) < 1$. The most generally used expression for the calculation of the confidence interval minimum mean life is, per Reference 1,

$$\overline{\log N_c} = \overline{\log N_i} - t_c\,(S_i/\sqrt{n}) \tag{13}$$

where,

n = sample size

$\overline{\log N_i}$ = sample mean

$= (\Sigma \log N_j)/n,\ j = 1, 2, 3 \ldots n$

S_i = sample standard deviation

$= [(\Sigma\ (\log N_j - \overline{\log N_i})^2)/(n - 1)]^{1/2}$

t_c = Student's t distribution t value for (n - 1) degrees of freedom and confidence c, Ref. 1, Table 29. (In Ref. 1, $c = \beta_2$).

Another expression, based on the concept of confidence region, and a joint estimate of the population mean and standard deviation, as defined in Reference 7, can be written as,

$$\overline{\log N_c} = \overline{\log N_i} - (m_{c_1} S_i)/[\sqrt{n}\ (\chi^2/df)^{1/2}_{c_2}] \tag{14}$$

where,

m_{c_1} = Number of standard deviations from mean corresponding to $(1-c_1)$ cumulative probability of failure; $|x|$ in Figure 32.

c_1 = Mean life confidence level

$(\chi^2/df)_{c_2}$ = (χ^2/df) values, Ref. 1, Table 30, corresponding to (n - 1) degrees of freedom for $100\ (1 - c_2)$ percentile

c_2 = standard deviation confidence level

$c = c_1 c_2$ = confidence level of the region

= joint probability of the mean and standard deviation confidence levels.

A third expression, based on a known population standard deviation, σ, can be written as,

$$\overline{\log N_c} = \overline{\log N_i} - m_c \sigma/\sqrt{n} \tag{15}$$

where,

m_c = Number of standard deviations from mean corresponding to (1 - c) cumulative probability of failure; $|x|$ in Figure 32.

This expression is based on the fact that if random samples are chosen from a Normal population, then the quantity

$$(\overline{\log N_i} - \overline{\log N})/(\sigma/\sqrt{n})$$

is Normally distributed with zero mean and a standard deviation of unity.

As an illustration of the mean life estimation by the three expressions, equations (13), (14) and (15), two typical aluminum alloy constant amplitude loading fatigue test samples are chosen:

	Sample 1	Sample 2
Ref	7	35
Specimen	Notched Sheet, K_t = 4	Riveted Lap Joint
n	13	3
$\overline{\log N_i}$	5.05	4.927
$\bar{N}_i$	112,000	84,500
S_i	0.335	0.091

Calculation of the expected population minimum life for the confidence level c = 0.95 gives the following results for the first sample:

Eq. (13), Student's t distribution,

$$\overline{\log N_c} = 5.05 - 1.78\ (0.335/\sqrt{13}) = 5.05 - 0.165 = 4.885$$

$$\bar{N}_c = 76{,}700 \text{ cycles}$$

Eq. (14), Confidence region, joint probability confidence,

$$c = c_1 \times c_2 = 0.975 \times 0.975 = 0.95$$

$$\overline{\log N_c} = 5.05 - (1.96 \times 0.335)/[(\sqrt{13})\sqrt{(0.367)}] = 5.05 - 0.301 = 4.749$$

$$\bar{N}_c = 56{,}100 \text{ cycles}$$

Use of Eq. (15) requires the knowledge of the population standard deviation, σ. If the σ values calculated from very large samples of test data, such as those presented by Figures 34 and 35 in the Appendix, can be assumed to be representative true population values, then the mean life estimates by Eq. (15) become,

$\sigma = 0.35$, Ref. Fig. 34, based on notched specimen data for $\bar{N}_i = 112{,}000$ cycles

$$\overline{\log N_c} = 5.05 - 1.65\,(0.35)/\sqrt{13} = 5.05 - 0.16 = 4.89$$

$$\bar{N}_c = 77{,}600 \text{ cycles}$$

or if, $\sigma = 0.29$, Ref. Fig. 38, based on combined unnotched and notched specimen data for $\bar{N}_i = 112{,}000$ cycles

$$\overline{\log N_c} = 5.05 - 1.65\,(0.29)/\sqrt{13} = 5.05 - 0.133 = 4.917$$

$$\bar{N}_c = 82{,}600 \text{ cycles}$$

Similarly, for the second sample, population mean life estimates for $c = 0.95$, by the three expressions are:

Eq. (13), Student's t distribution, $\bar{N}_c = 59{,}400$ cycles.

Eq. (14), Confidence region, joint probability confidence, $\bar{N}_c = 19{,}000$ cycles.

Eq. (15), Population standard deviation known ($\sigma = 0.14$ for structural components at $\bar{N}_i = 84{,}500$ cycles. Ref. Figure 38), $\bar{N}_c = 62{,}200$ cycles

If we tabulate the results of the two samples,

	Sample 1			Sample 2		
n S_i $\bar{N}_i$	13 0.335 112,000			3 0.091 84,500		
Eq.	σ	$\bar{N}_c$	$(\bar{N}_i/\bar{N}_c)$	σ	$\bar{N}_c$	$(\bar{N}_i/\bar{N}_c)$
13		76,700	1.46		59,400	1.42
14		56,100	2.00		19,000	4.45
15	0.35	77,600	1.44	0.14	62,200	1.36
15	0.29	82,600	1.36			

and define $(\bar{N}_i/\bar{N}_c) = SF|^c$ as the scatter factor for the population mean life estimation with respect to sample mean with confidence c, we observe that equations (13) and (15) define approximately the same population mean life estimates whereas equation (14) gives a rather conservative estimate. Of the three expressions, equations (13), (14) and (15), the strongest estimator is equation (15), provided the population standard deviation, σ, is known. Equation (14) is a weak and conservative estimator based on confidence interval estimates of the population minimum mean and maximum standard deviation values. Consequently, when the population standard deviation is known, such as the values presented for aluminum alloys in the Appendix, use of equation (15) is recommended for population mean life estimates. Use of equation (15) with the derived basic scatter distribution, equation (8), $m_c = |x|$ values, is also recommended. One other advantage of equation (15) is that mathematically the mean life estimate can be obtained from a sample size n = 1. When σ is not known, then equation (13) should be used for mean life estimation. This procedure of estimating the population mean life for a specified confidence level is recommended for the establishment of the median life S-N curves used for cumulative damage calculation and life prediction. When a structural element life is predicted analytically using the linear cumulative damage rule, the predicted life, corresponding to damage of 1.0, can be most correctly taken to represent the median life $\bar{N}_c$ with the confidence level c of the S-N data.

1.2 Scatter Factors with Respect to Sample Mean Life. Given a sample of size n and the sample mean life, $\bar{N}_i$, and standard deviation, S_i, as calculated by equations (4) and (5), the life N_{cp}, corresponding to the probability of failure, p, and confidence level, c, can be calculated in a number of different ways, similar to the estimation of the population mean life. Again, for comparison, three different expressions are presented for the calculation of N_{cp}, assuming that the sample comes from a Normally distributed population.

1. Based on the non-central t distribution, Table 33 in Reference 1, presents 'one-sided tolerance factor' k where,

$$k = f(n, p, c)$$

and

$$\log N_{cp} = \overline{\log N_i} - k_{cp} S_i \tag{16}$$

where in Reference 1, Table 33, p = percent survival and c = γ.

2. Based on the concept of the confidence region and a joint estimate of the population mean and standard deviation, as defined in Reference 7, using equation (14) for the population mean life estimate and the relationship of equation (10) with $\sigma = S_i/(\chi^2/df)^{1/2}_{c_2}$

$$\log N_{cp} = \overline{\log N_c} - m_p\sigma$$

$$= \overline{\log N_i} - [(m_{c_1}\ S_i)/\sqrt{n}\ (x^2/df)^{1/2}_{c_2}] - (m_p\ S_i)/(x^2/df)^{1/2}_{c_2}$$

$$= \overline{\log N_i} - [S_i/(x^2/df)^{1/2}_{c_2}]\ [m_{c_1}/\sqrt{n}) + m_p] \tag{17}$$

where,

$c = c_1 \times c_2$ and the other parameters as defined for equations (10) and (14).

3. If the population standard deviation is assumed to be known, then, using equation (15) for the population mean estimate, and the relationship of equation (10),

$$\log N_{cp} = \overline{\log N_c} - m_p\sigma$$

$$= \overline{\log N_i} - (m_c\sigma/\sqrt{n}) - m_p$$

$$= \overline{\log N_i} - \sigma[(m_c/\sqrt{n}) + m_p] \tag{18}$$

The scatter factors with respect to the sample mean, based on equations (16), (17), and (18) are,

$$SF|^c_p = (\bar{N}_i/N_{cp})$$

$$= \text{Antilog}\ (S_i k_{cp}) \tag{19}$$

$$= \text{Antilog}\ [S_i/(x^2/df)^{1/2}_{c_2}][m_{c_1}/\sqrt{n}) + m_p] \tag{20}$$

$$= \text{Antilog}\ \sigma\ [(m_c/\sqrt{n}) + m_p] \tag{21}$$

Table 2 presents scatter factors, based on the test data samples used for the mean life estimate illustration, as calculated by equations (19), (20) and (21). Similar to the population mean life estimate expression, equation (14), based on the confidence region concept, equation (20), based on the same concept, is a weak and unrealistically conservative expression for the calculation of basic fatigue scatter factors with respect to the sample mean. Equation (21) is the strongest and most general expression for the calculation of such scatter factors, provided, the population standard deviation, σ, is known. Therefore, when σ is known, such as the values for aluminum alloys presented in the Appendix, use of equation (21), together with the derived basic scatter distribution, equation (8), properties for m_c and m_p values, is recommended for the calculation of the basic fatigue scatter factors. Table 3 presents scatter factors calculated by equation (21), σ = 0.14, for selected values of n, c and p. For comparison purposes, the scatter factors were calculated on the basis of the Normal and test data derived, equation (8), distributions. The difference between the two distributions is clearly illustrated in Figure 3 for $c = 0.95$. The standard deviation of $\sigma = 0.14$ is a representative population standard deviation value for aluminum alloy notched specimen and structures under spectrum loading, see Appendix.

2. Fatigue Life Scatter Under Operating Conditions

Fatigue life scatter of a structural element in a fleet of aircraft, in addition to the basic fatigue scatter, is also a function of the applied loads and environment variation between individual aircraft. No two aircraft experience 'identical' loadings or environments. Thus, the probability of failure of a structural element in a fleet of aircraft is a function of two variables:

1. Basic Fatigue Scatter - N
2. Applied Loads - Environment Variation - L

Consequently, the probability of failure of a structural element in a fleet of aircraft at a specified life N_j is a joint probability distribution function of two dependent variables:

$$p(N_j) = \sum_i p(N_j L_i) = \sum_i p(N_j|L_i) \times p(L_i) \tag{22}$$

where, $p(N_j|L_i)$ = probability of failure at N_j given L_i

$$= p(N_j L_i)/p(L_i) \tag{23}$$

= basic fatigue scatter

$p(L_i)$ = probability of occurrence of L_i

= applied loads - environment variation.

Then the cumulative probability of failure at a specified life N_j, i.e., the probability of failure in the life interval $0 < N \leq N_j$, is:

$$P(N_j) = \sum_j p(N_j) \tag{24}$$

The concepts of a joint probability distribution and the calculation of operational life scatter factors are illustrated in Figure 4. Here, the concept is presented for the discrete case where the probability $p(L_i)$ represents the probability of experiencing load spectrum L_i, where L_i may represent an average load spectrum over a discrete interval ΔL_i, and the probabilities $p(N_j|L_i)$, $p(N_j L_i)$ and $p(N_j)$ represent the probability of failure over a discrete life interval ΔN_j. The calculation of the operational life scatter factors consists of five basic steps:

1. Definition of the applied loads spectrum probability distribution, $p(L_i)$, where L_i is a measure of the spectrum magnitude.

2. Calculation of life probability distribution for each L_i spectrum, $p(N_j|L_i)$. The procedure consists of calculating the mean life, $\bar{N}_i$, for each specified applied loads spectrum, L_i, and then calculating the probability distribution with respect to the mean, using an acceptable basic fatigue scatter distribution. Calculate the $p(N_j|L_i)$ values for each distribution corresponding to the same N_j interval.

3. Calculation of the joint probability distribution, $p(N_jL_i) = p(N_j|L_i) \times p(L_i)$.

4. Calculation of the operational life probability distribution, $p(N_j) = \sum_i p(N_jL_i)$.

5. Calculation of the operational life scatter factors,

$$SF|_p^c = (\bar{N}_c/N_p) \tag{25}$$

where, $\bar{N}_c$ = Mean operational life corresponding to $\Sigma p(N_j) = .5$

p = Probability of failure, corresponding to a specified life N_j, from the cumulative probability distribution, $\Sigma p(N_j)$.

c = Confidence level of the basic S-N data used in the life prediction in Step 2.

The cumulative probability distribution $\Sigma p(N_j)$ can be obtained directly in step 4 by calculating the conditional distributions $p(N_j|L_i)$ in step 2 as cumulative probabilities.

The unknown in this problem is the $p(N_j)$ marginal distribution, given the applied loads, $p(L_i)$, and the corresponding life, $p(N_j|L_i)$ distributions. However, if it can be assumed that life prediction for a specified loads spectrum, L_i, is possible and the basic fatigue life scatter distribution is known, the real unknown of the problem is the applied loads distribution, $p(L_i)$. A truly statistical treatment of the applied loads spectra variation among individual aircraft in a fleet of aircraft is almost nonexistent. However, a recent paper by Bouchard, Reference 3, indicates a growing interest in the area of individual aircraft applied loads spectra, and it is hoped that in the future the appropriate agencies collecting operational loads data will evaluate and present the data in terms of individual aircraft experiences. From such data, it would be possible to construct applied loads probability distribution models for specified types of aircraft and missions, or a mix of missions that a certain type of aircraft would be expected to perform. A complete definition of the operational loads spectrum should include at least:

1. Incremental loads spectrum frequency and magnitude.

2. Operational 1.0g loads magnitude and frequency.

3. Landing frequency.

It must be also noted, as an obvious conclusion from the above discussion, that for most aircraft and structural elements, 'flight hours' is not the absolute measure of the fatigue life. Life measure in terms of 'flight hours' must be always associated with the various applied loads spectrum parameters, which in essence define the life of the structural element.

For the purpose of illustration, Figure 5 presents a joint probability distribution model based on the following assumptions:

1. Applied loads spectrum distribution, $p(L_i)$ is Normal.

2. Conditional life distributions, $p(N_j|L_i)$ are log Normal and have the same log standard deviation, $\sigma_{(N|L)}$.

3. The mean life $\overline{\log N_i}$, of $p(N_j|L_i)$ distributions varies linearly with L_i, where $\overline{\log N_{i+1}} = \overline{\log N_i} - \sigma_{N|L}$. This is a purely hypothetical assumption and in retrospect defines the magnitude of L_i values. In real problems, $\bar{N}_i = f(L_i)$.

4. The $p(L_i)$ and $p(N_j|L_i)$ distributions were truncated at $\mu \pm 3.5\sigma$.

Because of the assumptions made in constructing the probability model of Figure 5, the resulting joint distribution is a Bivariate Normal Distribution and the marginal life distribution, $p(N)$ is also Normal. The subject of the Bivariate Normal Distribution is discussed in Reference 4 by Hoel. The important properties of the Bivariate Normal Distribution are: the marginal, conditional, and the joint distributions are Normal, all conditional distributions have the same standard deviation, and the mean of the conditional distributions varies linearly. All of these properties must be met if the marginal life distribution $p(N)$ is to be Normal. However, in most realistic operational life probability problems all properties of the Bivariate Normal Distribution will not be satisfied, principally, the normality of the $p(L)$ distribution and the linear variation of the mean of the $p(N|L)$ distributions. As stated earlier, the marginal life distribution $p(N)$ of Figure 5 is log Normal and the resulting properties of the distribution and the operational life scatter factors can be calculated in the following manner:

1. The scatter factors, $SF|_p^c = \bar{N}_c/N_p$, can be directly calculated from the marginal $p(N)$ distribution, where

$\bar{N}_c$ = Marginal distribution mean life associated with the confidence level, c, of the basic S-N data used in calculating the conditional distribution mean lives, N_i.

$$N_p = \text{Antilog}\,[\overline{\log N_c} - m_p'(\sigma_{N|L})] \tag{26}$$

where, m_p' = Number of $\sigma_{N|L}$ standard deviations from the mean, $\bar{N}_C$, corresponding to the probability of failure $p = \Sigma p(N_j)$ in Figure 5.

$$= \left|\frac{\log N_j - \overline{\log N_C}}{\sigma_{N|L}}\right|$$

and, according to equation (11),

$$SF|_p^C = (\bar{N}_C/N_p) = \text{Antilog}\,(|m_p'|\sigma_{N|L}),\ N_p < N_C$$

This is a general expression for operational life scatter factors when the joint probability function is Bivariate Normal. It should be noted that m_p' refers to the conditional distribution, $p(N|L)$, standard deviation $\sigma_{N|L}$, and not to the marginal life distribution $p(N)$ standard deviation σ_N.

2. The standard deviation, σ_N, of the $p(N)$ distribution can be calculated from the general properties of the Bivariate Normal Distribution as presented in Reference 4:

$$\sigma_N = \sigma_{N|L}\ (\sqrt{1 - \rho^2}) \tag{27}$$

where, $\rho = \sigma_{NL}/\sigma_N\sigma_L$ = correlation coefficient (28)

σ_{NL} = Covariance of the joint distribution

$$= \sum_j \sum_i (N_j - \bar{N})\ (L_i - \bar{L})\ p(N_jL_i) \tag{29}$$

However, σ_N can be easier calculated by the expression,

$$\sigma_N = (m_p'\ \sigma_{N|L})/m_p \tag{30}$$

Where,

m_p' = Number of $\sigma_{N|L}$ from the mean, $\bar{N}$, corresponding to $p = \Sigma p(N_j)$ in the marginal life distribution in Figure 5.

m_p = Number of σ from the mean of a Normal distribution corresponding to $p = \Sigma p(N_j)$. This value can be obtained from Figure 32 in the Appendix, $m_p = |x|$.

For the joint distribution of Figure 5, for $p = \Sigma p(N_j) = 0.0415$, $m_p' = 2.5$ and $m_p = 1.75$, and therefore,

$$\sigma_N = 2.5\ \sigma_{N|L}/1.75 = 1.43\ \sigma_{N|L} \tag{31}$$

Thus, for the assumed p(L) distribution and the resulting $\bar{N}_i$ variation, this expression for σ_N is valid for any $\sigma_{N|L}$ value which is constant for all p(N|L) distributions. Consequently, the operational life scatter factors in terms of σ_N can be calculated as,

$$SF|_{N}^{C} = (\bar{N}_c/N_p) = \text{Antilog}\,(|m_p|\sigma_N) \quad , \quad N_p < \bar{N}_c \tag{32}$$

$$= \text{Antilog}\,(1.43|m_p|\sigma_{N|L})$$

In the fatigue test data survey, as presented in the Appendix, $\sigma_{N|L} = 0.14$ was found to be representative of the basic fatigue life scatter of notched specimen and structures under spectrum loadings; also a $\sigma = 0.20$ was calculated for the test life scatter under spectrum loadings of full-scale structures which had experienced previous service loadings, and thus, the value of $\sigma = 0.20$ reflects not only the basic fatigue scatter, but also the variability of applied loads spectrum of individual aircraft. It is interesting to note that for the joint distribution of Figure 5, for a value of $\sigma_{N|L} = 0.14$, $\sigma_N = 1.43$ $(0.14) = 0.20$. This apparent correlation of the two values with the test data survey results can be considered to be coincidental, since the joint distribution was based on purely hypothetical assumptions. Nevertheless, it indicates that the concept of the operational life scatter as a function of the joint probability distribution of the basic fatigue scatter and applied loads variation is a realistic approach for the establishment of operational life scatter factors. The values of the scatter factors for $\sigma_N = .20$ of a Normal distribution can be directly read from the $\sigma = .20$ curve of Figure 2.

If the Normal conditional life distribution, p(N|L), in Figure 5 is replaced by the basic fatigue scatter distribution derived from fatigue test data, equation (8), $\sigma_{N|L} = .14$, the resulting joint and marginal life distributions are shown in Figure 6. The resulting operational life scatter factors from the two joint distributions, Figures 5 and 6, $\sigma_{N|L} = .14$ are shown in Figure 9. The probabilities of failure of the two distributions for selected scatter factors are:

$SF = \bar{N}/N_p$	Σ Probability of Failure - %	
	Bivariate Normal	p(L) - Normal p(N|L) - Test Data, Eq. (8)
1.5	19.0	17.0
2.0	6.7	6.4
3.0	.83	1.2
4.0	.13	.46
5.0	.02	.25

As a final illustration of the operational fatigue life scatter joint probability distribution concept, a military transport aircraft service failure history case was considered. In the course of fatigue analysis of this aircraft, Reference 5, service records indicated that the utilization of the aircraft, as it affects fatigue life, varied greatly for certain groups of aircraft. All aircraft were divided into five groups according to their average utilizations and five different loads spectra were defined for the five groups. Table 4 presents a general description of the five utilizations and the resulting predicted mean lives for the wing spar cap element at a structural discontinuity. Figures 7 and 8 show the joint probability and marginal life distributions based on the applied loads distribution, $p(L_i)$, and the mean lives, $\bar{N}_i$, of Table 4. Both distributions are based on $\sigma_{N|L} = .14$; however, Figure 7 is based on $p(N|L)$ Normal, while Figure 8 $p(N|L)$ distribution is the test data distribution, equation (8). The joint distributions are not shown for lives $N > 30{,}000$ flight hours since the main interest lies in lives shorter than the mean. The resulting scatter factors of the two distributions are shown in Figure 9. The most interesting aspect of these operational life scatter distributions is their comparison to the wing spar cap service failure history. When the fleet of approximately 395 aircraft were inspected for fatigue cracks in the wing spar cap, 44 of the subject elements were found to contain cracks of various lengths. At the time of inspection, the fleet average flight time was approximately 11,500 flight hours. Individual aircraft flight time ranged from approximately 7,000 to 18,000 flight hours. Table 5 presents the flight time history of the aircraft at inspection and the service and predicted failure distributions. A fairly good agreement exists between the predicted and the actual total number of service failures: 39 predicted versus 44 actual failures. The failure probability distributions, as shown in Figure 10 exhibit good agreement between predicted and actual failures in view of the accuracy of fatigue analysis life prediction and lack of detail information about service failure crack lengths. It is to be noted that the theoretical probability distributions predict visible crack initiation whereas numerous service cracks had propagated beyond this stage. Thus, in view of the fact that a number of service cracks must have initiated at an earlier time than they were discovered during the particular fleet inspection, the probability distribution of Figure 10, based on $p(N|L)$ test data distribution, is considered to be a valid representation of the fatigue crack initiation life distribution. Typical scatter factors and associated probabilities of failure for this operational life distribution, see Figure 9, are:

$SF = \bar{N}/N_p$	$\Sigma p(N_j)$ - %
2.0	4.1
3.0	0.85
4.0	0.37

In conclusion, it appears that the operational life probability distribution, based on the joint probability distribution of the basic fatigue scatter and applied loads variations is a valid concept, and perhaps the most promising concept in defining operational life requirements for fatigue analysis and design of aircraft structures. If an operational life joint probability distribution model can be constructed, as illustrated by Figure 4, then all of the probability of failure information about a fleet of aircraft is completely defined:

$\Sigma p(N_j)$ - cumulative probability of failure in a fleet of aircraft at time N_j, i.e., $\Sigma p(N_j)$ specifies the proportion of the fleet that can be expected to initiate a fatigue crack in a structural element under consideration in the time interval, $0 < N \leq N_j$.

$\Sigma_j p(N_j|L_i)$ - cumulative probability of failure at time N_j of an aircraft, or a group of aircraft, given that the aircraft experience the applied loads spectrum L_i.

$\Sigma_j p(N_j L_i)$ - cumulative probability of failure at time N_j in a fleet of aircraft due to spectrum L_i with the associated probability $p(L_i)$.

$p(N_j)$, $p(N_j|L_i)$, $p(N_j L_i)$ - probabilities of failure, as defined above, during the time interval $N_j = \Delta N$.

$\Sigma p(N_j) = .50$ Specifies the median operational life of the fleet, i.e., it is expected that half of the structural elements under consideration in a fleet of aircraft would experience fatigue failures, crack initiation, by the time the fleet reaches life $\bar{N} = N_j$.

It is extremely questionable whether a single joint distribution can be derived to represent the operational life distribution of any fleet of aircraft. The operational life distribution is a function of the applied loads spectrum variation within a fleet of aircraft, and this variation is not necessarily identical for all types of aircraft. It is probable that a study of the applied loads spectrum variation of many types of aircraft would indicate a standardization of the p(L) distribution for different types of aircraft, and consequently, standard p(L) distributions could be used in the fatigue design and analysis of any fleet of aircraft.

2.1 <u>Mean Operational Life</u>. The concept of the mean, or more properly, the median, service operational life estimate of a structural element for a fleet of aircraft is self evident in the operational life joint probability distribution presentation in this section. The median life, $\bar{N}_c$, is the N_j value which corresponds to $\Sigma p(N_j) = .50$ in the marginal p(N) distribution. The confidence level corresponds to the confidence level of the S-N data used in calculating the mean lives, $\bar{N}_j$, of the conditional life, $p(N_j|L_i)$, distributions. It is obvious that this does not reflect the confidence level assigned, if any, to the $p(L_i)$ distribution. However, if a confidence level is defined for the $p(L_i)$ distribution, then the operational median life confidence level, C_N, would correspond to the joint probability of the two confidence levels, $C_N \times C_{N|L}$. It is to be noted that the median operational life does not necessarily correspond to the mean or average applied loads spectrum, $\bar{L}$. Thus, prediction of the mean life on the basis of average utilization spectrum does not necessarily imply that the predicted life is the median or mean operational life.

3. Operational Life Scatter Factors and Fleet Size

In the preceding discussions of the basic and operational fatigue life scatter, the frequency and probability distributions were defined for populations approaching infinity in size. However, when dealing with aircraft fleet sizes, the sizes are finite and generally will range from 50 to 1,000 aircraft. If a structural element in a fleet of size n was allowed to fail in all aircraft and the time of each failure was noted, then, by arranging the time to failure in increasing order, the failure distribution can be plotted as

$$F(N_j) = \Sigma n_j/(n+1) \quad , \quad j = 1,2,3...n \tag{33}$$

The life of the first failure, N_1, can be related to the mean life of all failures, $\bar{N}$, in the form of a scatter factor, $SF|_p = \bar{N}/N_1$, where $p = F(N_1)$ from equation (33). Thus, if $F(N_j)$ distribution is compared with the population probability of failure distribution, then the theoretically calculated scatter factor, $SF|_p$, for a probability of failure $p = F(N_1)$, would define the time to first failure.

It is obvious that for symmetrical aircraft structures there are two identical structural elements per airplane. Thus for symmetrical structures, the sample size which must be statistically evaluated is twice the fleet size. Consequently, reference to a fleet of size n implies the sample size of all identical structural elements, where the word 'identical' means identically designed and loaded elements.

Table 6 presents scatter factors for the time to first failure as calculated for different fatigue life distributions in this report and as calculated by Freudenthal in Reference 6 for $\sigma_{N|L}$ = .14 and fleet size n = 20 to 1,000. The scatter factors, as calculated in this report, are shown for the Normal and test data derived distributions for the basic fatigue scatter and operational life joint distribution models. For comparison, scatter factors are also presented for the basic fatigue scatter model based on an estimate of the mean life with 95% confidence from test data sample of n = 3. Here, as in earlier comparisons of the Normal and test data derived distributions, the Normal distribution, in general, results in unconservative scatter factors for the time to first failure. It is interesting to note that the scatter factors based on the derived test data distribution of this report and those of Reference 6, although based on different distributions and basic data, are similar. The first time to failure scatter factors vary approximately from 2 to 3 for fleet sizes 20 < n < 100, SF = 3 to 4 for 100 < n < 200, and SF = 4 to 8 for 200 < n < 1000.

The similarity between the scatter factors of Table 5 based on the basic fatigue scatter model of this report and those of Reference 6 are not surprising since both values are based on somewhat similar extremal value distributions. However, the similarity of the joint distribution scatter factors of this report and those of Reference 6 could be viewed as coincidental, since the joint distribution factors are a function of the applied loads spectrum distribution which can vary for different fleets of aircraft. Therefore, the joint distribution concept appears to be the most realistic approach for the calculation of the first time to failure scatter factors for a given fleet of aircraft.

4. Scatter Factors and Design Life Requirements

The following procedure is recommended for the specification and verification of fatigue life design requirements:

1. Specify the required life, N_R, where, $R = (1-p)$, is the desired reliability and p is the probability of failure at time N_R.

2. Define the expected fleet utilization in terms of mission profiles.

3. By analysis and/or testing establish the fleet mean (or median) life $\bar{N}$ for a desired confidence level, c.

4. Calculate the scatter factor $SF|_p$ for the specified probability of failure, p.

5. Calculate the life, N_p, corresponding to the specified probability of failure, p, as: $N_p = \bar{N}/(SF|_p)$. When the life estimate is directly based on the structural element test results, where the test spectrum represents the mean life environment, steps (3) and (4) can be combined by calculating N_p directly from the test sample mean life, $\bar{N}_1$, in conjunction with $SF|_p^c = (\bar{N}_1/N_{cp})$. Samples of these scatter factors are tabulated in Table 3.

6. Calculate the fatigue life margin of safety as,

$$MS_{FL} = (N_p/N_R) - 1 \tag{34}$$

7. A $MS_{FL} \geq 0$ indicates that the design life requirement has been satisfied. If $MS_{FL} > 0$, the probability of failure at the required life is less than the specified value and it corresponds to the probability of failure associated with $SF = (\bar{N}/N_R)$. Also, subject to other strength requirements, a $MS_{FL} > 0$ indicates that structural weight can be reduced by increasing the design stress of the structural element to a level which would result in $MS_{FL} = 0$.

8. A $MS_{FL} < 0$ indicates that the design life requirement has not been fulfilled. The structural element must be redesigned by improving its fatigue quality and/or by reduction of the design stress level.

In the above outline of the fatigue life design criteria the aspect of the desired reliability for the specified design life requires further clarification and discussion. Two approaches can be taken in specifying the desired reliability. One is the concept of fleet size and the time to first failure. The other approach is to specify a general reliability level regardless of the fleet size. Since scatter factors are directly related to the reliability, or more properly, probability of failure, p, the difference between the two approaches can be illustrated by looking at the scatter factors for the time to first failure from Table 6:

Fleet Size n	p ~ % 100/(n+1)	$SF\|_p$
20	4.76	1.90
50	1.96	2.40
100	.99	2.85
200	.5	3.60
1000	.1	7.70

It is seen that if the time to first failure concept is used in specifying the design life reliability requirements, a relatively high probability of failure is accepted for small fleet sizes, whereas, for large fleet sizes the scatter factors become high and result in extremely long mean life requirements. For example, for a sample size of 100 the time to first failure corresponds to life ($\bar{N}/2.85$) and for sample of 1,000 the time to first failure corresponds to life ($\bar{N}/7.7$). If the required life was specified to be N_R = 30,000 flight hours, then the design for a sample of 100 would require a mean life $\bar{N}$ = 30,000 x 2.85 = 85,500 flight hours and for sample of 1000, $\bar{N}$ = 30,000 x 7.7 = 231,000 flight hours. Thus, using this approach, the requirements vary greatly as a function of the fleet size. However, fleet sizes as defined in the design stages often, at a later date, change and increase. Thus, rigid adherence to this rule will not always be possible or practical. Consequently, the designer would tend to reduce the probability of failure for the required life below the level of the first time to failure on the basis of design stage fleet size estimate. Of course, this leads toward the other approach of specifying a generally acceptable reliability level regardless of fleet size. In conclusion, it appears that the procuring agency should specify a general reliability level on the basis of aircraft type and its operational requirements. In conjunction with an increase in inspection frequency after the time to first failure, a probability of failure, p, from .2 to 2.0% with a 90 or 95% confidence on the mean life estimate appears to on a realistic reliability range to consider in specifying design life requirements. The scatter factors, with respect to the mean life for this range of probabilities of failure vary from approximately 2.5 to 5.5, see Table 6 and Figures 3 and 9. In the past the scatter factors most commonly used have been 2, 3, and 4. It is interesting to note the probabilities of failure associated with these factors as determined in this study and Reference 6. For $\sigma_{\bar{N}|L}$ = .14 and test data derived basic scatter distribution, the probabilities of failure are:

	p ~ %		
SF =	2	3	4
Basic Scatter, n = ∞ ; Fig. 2	2.2	.55	.28
Basic Scatter, n = 3, c = .95; Fig. 3	7.8	1.42	.55
Joint Distribution, n = ∞ ; Fig. 9: Hypothetical	6.5	1.2	.46
Transport	4.1	.85	.37
Ref. 6, n = ∞	7.0	1.4	.5

For general purposes it may be stated that operational life scatter factors of 2, 3, and 4 correspond to approximately 6.0, 1.0 and .5% probability of failure.

TABLE 1

FATIGUE LIFE PROBABILITIES OF FAILURE

X=Number of σ from the Mean	Σ Probability of Failure - %							
	Based on Test Data - Equation (8)							Normal Distribution
	σ=	.05	.10	.14	.20	.50	.75	
-9.0		.05	.035	.027	.021	.007	.004	
-7.0		.1	.080	.067	.053	.024	.015	
-6.0		.16	.13	.11	.089	.043	.029	
-5.0		.25	.20	.18	.15	.085	.063	
-4.0		.45	.38	.34	.31	.20	.14	< .005
-3.0		1.00	.88	.83	.78	.61	.51	.13
-2.5		1.70	1.52	1.47	1.43	1.26	1.12	.62
-2.0		3.09	2.80	2.76	2.71	2.71	2.62	2.3
-1.5		5.90	6.42	5.49	5.53	6.04	6.25	6.7
-1.0		11.8	11.3	11.2	11.5	13.6	16.1	15.99
- .5		24.1	23.5	23.6	24.0	27.9	31.4	30.9
0		50.0	50.0	50.0	50.0	50.0	50.0	50.0
+ .5		75.9	76.5	76.4	76.0	72.1	63.6	69.1
1.0		88.2	89.7	89.8	89.5	86.4	83.9	84.1
1.5		94.1	93.58	94.51	94.47	93.96	93.75	93.3
2.0		96.91	97.2	97.24	97.29	97.29	97.38	97.7
2.5		98.3	98.48	98.53	98.57	98.74	98.88	99.38
3.0		99.0	99.12	99.17	99.22	99.39	99.49	99.87
4.0		99.55	99.62	99.66	99.69	99.80	99.86	>99.995
5.0		99.75	99.80	99.82	99.85	99.915	99.937	
6.0		99.84	99.87	99.89	99.911	99.957	99.971	
7.0		99.89	99.92	99.933	99.947	99.976	99.985	
9.0		99.95	99.965	99.973	99.979	99.993	99.996	

TABLE 2

EXAMPLES OF BASIC FATIGUE SCATTER FACTORS WITH RESPECT TO THE TEST SAMPLE MEAN LIFE

Sample 1:
$K_t = 4$, Edge-notch Al. Alloy Specimen

$n = 13$

$\bar{N}_i = 112{,}000$ cycles

$S_i = 0.33$[illegible]

Ref. 7

Sample 2:
Al. Alloy Riveted Lap Joint

$n = 3$

$\bar{N}_i = 84{,}500$ cycles

$S_i = 0.091$

Ref. 35

Constant Amplitude Loading

$c = .95$	$SF\|_p^c = (\bar{N}_i/N_{cp})$					
	Sample 1			Sample 2		
p=%	5.0	1.0	0.1	5.0	1.0	0.1
Normal Distrib. Eq. (19)	7.81	16.8	40	4.96	9.10	18.2
(20) $C_1 = C_2 = .975$	16.10	38.3	100	38.4	94.5	965
(21) $\sigma = 0.35$	5.41	9.4	17.4	—	—	—
$= 0.29$	4.05	6.4	10.6	—	—	—
$= 0.14$	—	—	—	2.30	2.87	3.66
Test Data Distrib. Eq. (8)						
Eq. (21) $\sigma = 0.35$	5.18	12.60	87	—	—	—
$= 0.29$	3.91	8.26	50	—	—	—
$= 0.14$	—	—	—	2.22	3.30	9.86

σ values taken from [illegible] 34 and 38

TABLE 3

FATIGUE LIFE BASIC SCATTER FACTORS WITH RESPECT TO THE TEST SAMPLE MEAN LIFE

a) Basic Fatigue Life Scatter Distribution, Eq. (8)

p=%	.1	.5	1	5	10	.1	.5	1	5	10
n	c = .85					c = .90				
1	9.53	3.99	3.18	2.14	1.84	10.5	4.4	3.52	2.36	2.03
3	8.55	3.58	2.87	1.92	1.65	9.04	3.78	3.04	2.03	1.74
5	8.36	3.46	2.77	1.86	1.60	8.64	3.62	2.88	1.94	1.67
10	8.00	3.34	2.68	1.80	1.54	8.20	3.45	2.76	1.85	1.59
20	7.80	3.27	2.61	1.75	1.51	7.96	3.34	2.68	1.79	1.54
50	7.64	3.20	2.56	1.72	1.48	7.74	3.24	2.60	1.74	1.50
	c = .95					c = .99				
1	12.2	5.11	4.09	2.75	2.36	18.2	7.62	6.11	4.09	3.52
3	9.86	4.13	3.30	2.22	1.90	12.4	5.20	4.16	2.80	2.40
5	9.23	3.86	3.10	2.08	1.78	11.1	4.62	3.70	2.48	2.13
10	8.63	3.62	2.90	1.94	1.67	9.81	4.10	3.29	2.20	1.89
20	8.25	3.45	2.77	1.85	1.59	9.0[illegible]	3.77	3.02	2.03	1.74
50	7.90	3.31	2.65	1.78	1.53	8.35	3.50	2.81	1.88	1.62
n=∞	7.36	3.08	2.47	1.66	1.42					

b) Normal Distribution

p=%	.1	.5	1	5	10	.1	.5	1	5	10
n	c = .85					c = .90				
1	3.78	3.20	2.95	2.37	2.11	4.08	3.46	3.19	2.56	2.28
3	3.28	2.78	2.56	2.06	1.83	3.43	2.90	2.68	2.15	1.91
5	3.13	2.66	2.45	1.97	1.75	3.25	2.76	2.54	2.04	1.82
10	3.00	2.54	2.34	1.88	1.68	3.08	2.61	2.41	1.93	1.72
20	2.92	2.47	2.28	1.83	1.63	2.96	2.51	2.32	1.86	1.66
50	2.84	2.40	2.22	1.78	1.58	2.86	2.43	2.24	1.80	1.60
	c = .95					c = .99				
1	4.59	3.89	3.58	2.88	2.56	5.71	4.84	4.46	3.59	3.19
3	3.66	3.12	2.[illegible]7	2.30	2.05	4.16	3.53	3.26	2.62	2.32
5	3.37	2.86	2.64	2.12	1.89	3.78	3.20	2.96	2.37	2.11
10	3.16	2.68	2.48	1.99	1.77	3.43	2.90	2.68	2.15	1.91
20	3.02	2.56	2.36	1.90	1.69	3.20	2.71	2.50	2.00	1.78
50	2.90	2.46	2.26	1.82	1.62	3.00	2.55	2.35	1.88	1.67
n=∞	2.70	2.29	2.11	1.70	1.51					

$SF\Big|_p^c = (\bar{N}_1/N_{cp})$, calculated by Eq. (21), $\sigma = .14$

$\bar{N}_1$ = sample mean life

c = confidence level, singular limit (one sided)

TABLE 4

TRANSPORT AIRCRAFT APPLIED LOADS SPECTRA
AND STRUCTURAL ELEMENT FATIGUE LIFE

Applied Loads Spectrum		Aircraft Utilization		No. of Aircraft, n_i	$p(L_i) = \frac{n_i}{\Sigma n}$	Wing Spar Cap Predicted Mean Life - $\bar{N}_i$ Flt. Hrs.
i	L_i	Missions	Flt. Hrs/Landg.			
1	IV	Service-Normal Trng.	5.54	42	.106	29,000
2	III	"	4.53	104	.264	26,300
3	II	"	4.0	77	.195	
4	I	"	2.75	145	.366	24,300
5	V	Service-High Trng.	1.62	27	.069	14,000

$\Sigma n = 395$

TABLE 5

TRANSPORT AIRCRAFT FLIGHT TIME AND FAILURE DISTRIBUTIONS

j	Flt. Hrs. $N_j \times 10^{-3}$	No. of Acft.	No. of Specimen(1) with $N > N_j$ (1)	Service Failures (2)			Predicted Failures (3)					
							p(N|L) Test Data			p(N|L) Normal		
				n	$p(\Delta N)$	$\Sigma p(N_j)$	$\Sigma p(N_j)$	$p(\Delta N)$	n	$\Sigma p(N_j)$	$p(\Delta N)$	n
1	0						0			0		
2	6.5	0	790	0	.0000	.0000	.0038	.0038	3.00	.0006	.0006	.47
3	7	12	790	2	.0025	.0025	.0043	.0005	.40	.0010	.0004	.32
4	8	55	766	5	.0065	.0090	.0069	.0026	1.99	.0026	.0016	1.23
5	9	46	656	12	.0183	.0273	.0101	.0032	2.10	.0059	.0033	2.16
6	10	50	564	0	.0000	.0273	.0150	.0049	2.76	.0114	.0055	3.10
7	11	49	464	6	.0129	.0402	.022	.007	3.25	.019	.0076	3.53
8	12	12	366	1	.0027	.0429	.032	.010	3.66	.029	.010	3.66
9	13	16	342	2	.0058	.0487	.046	.014	4.79	.043	.014	4.79
10	14	26	310	5	.0161	.0648	.063	.017	5.27	.061	.018	5.58
11	15	51	258	5	.0194	.0842	.083	.020	5.16	.083	.022	5.68
12	16	35	156	5	.0321	.1163	.102	.019	2.96	.110	.027	4.21
13	17	18	86	1	.0116	.1279	.122	.020	1.72	.140	.030	2.58
14	18	21	50	0	.0000		.145	.023	1.15	.17[illegible]	.034	1.70
15	19	4	8	0	.0000		.172	.027	.22	.210	.036	.29
				44					38.43			39.30

(1) Two specimen (wings) per aircraft.
(2) n-Number of failures - cracked Spar caps; $p(\Delta N) = n/$(1).
(3) Ref. Fig. 10 for $\Sigma p(N_j)$ values, where $p(\Delta N) = \Sigma p(N_j) - \Sigma p(N_{j-1})$ and $n = p(\Delta N) \times$ (1).

TABLE 6

SCATTER FACTORS FOR TIME TO FIRST FAILURE

Fleet (Sample) Size n	p ~ % 100/(n+1)	Ref. 6 Fig. 3	Mean Life Known						Mean Life Estimated from Sample n=3; c=.95	
			Fig. 2 Basic Scatter		Fig. 9 Oper. Life Joint Distrib.				Fig. 3 Basic Scatter	
			Normal(1)	Test Data(2)	Hypothetical(3) N	Hypothetical(3) TD	Transport(4) N	Transport(4) TD	N(1)	TD(2)
20	4.76	2.2	1.71	1.68	2.19	2.19	1.90	1.[illegible]	2.30	2.25
50	1.96	2.7	1.95	2.05	2.60	2.65	2.27	2.40	2.13	2.78
100	.99	3.3	2.11	2.47	2.90	3.15	2.60	2.85	2.87	3.30
200	.5	4.0	2.29	3.08	3.30	3.9	2.90	3.60	3.12	4.13
1000	.1	6.5	2.70	7.36	4.10	7.9	3.65	7.70	3.66	9.86

Population $\sigma = .14 = \sigma_{N/L}$

(1) Normal probability distribution, $p(N|L)$
(2) Probability distribution derived from test data, Eq. (8), $p(N|L)$
(3) Hypothetical applied loads, $p(L)$, probability distribution
(4) Actual transport aircraft $p(L)$ distribution

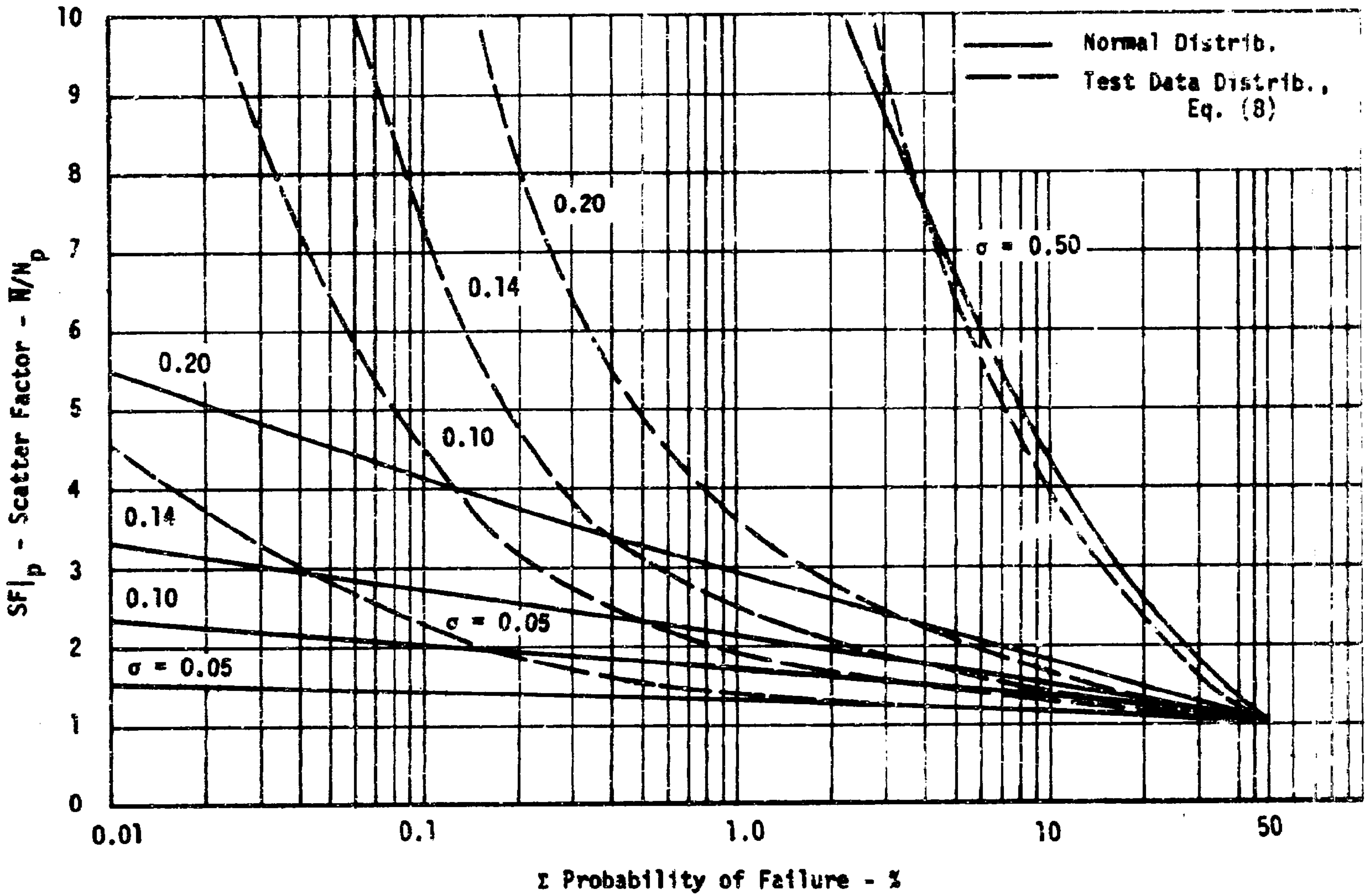


FIGURE 2. FATIGUE LIFE BASIC SCATTER FACTORS WITH RESPECT TO A KNOWN MEAN LIFE.

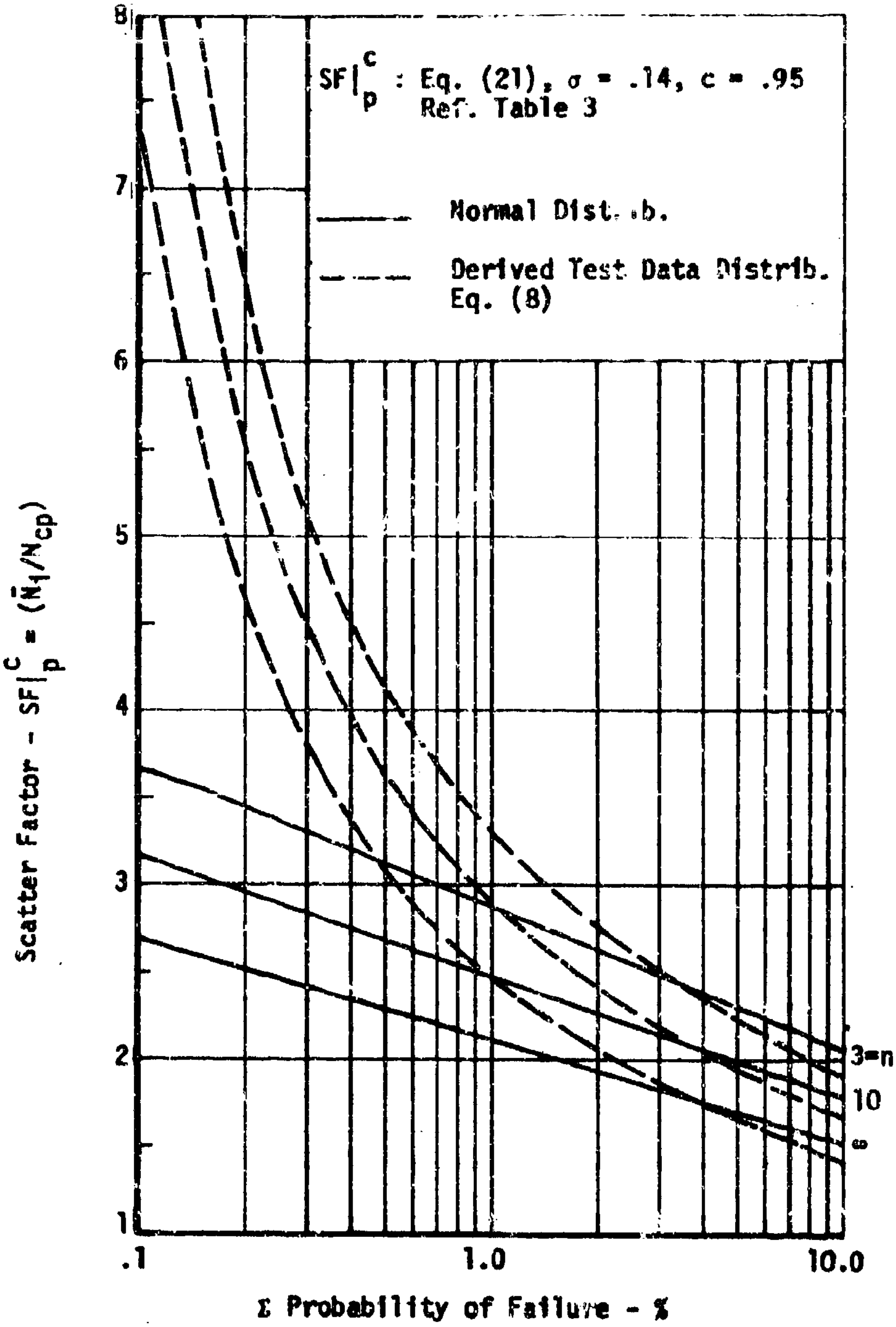


FIGURE 3. COMPARISON OF BASIC SCATTER FACTORS WITH RESPECT TO SAMPLE MEAN BASED ON THE NORMAL AND DERIVED TEST DATA DISTRIBUTIONS.

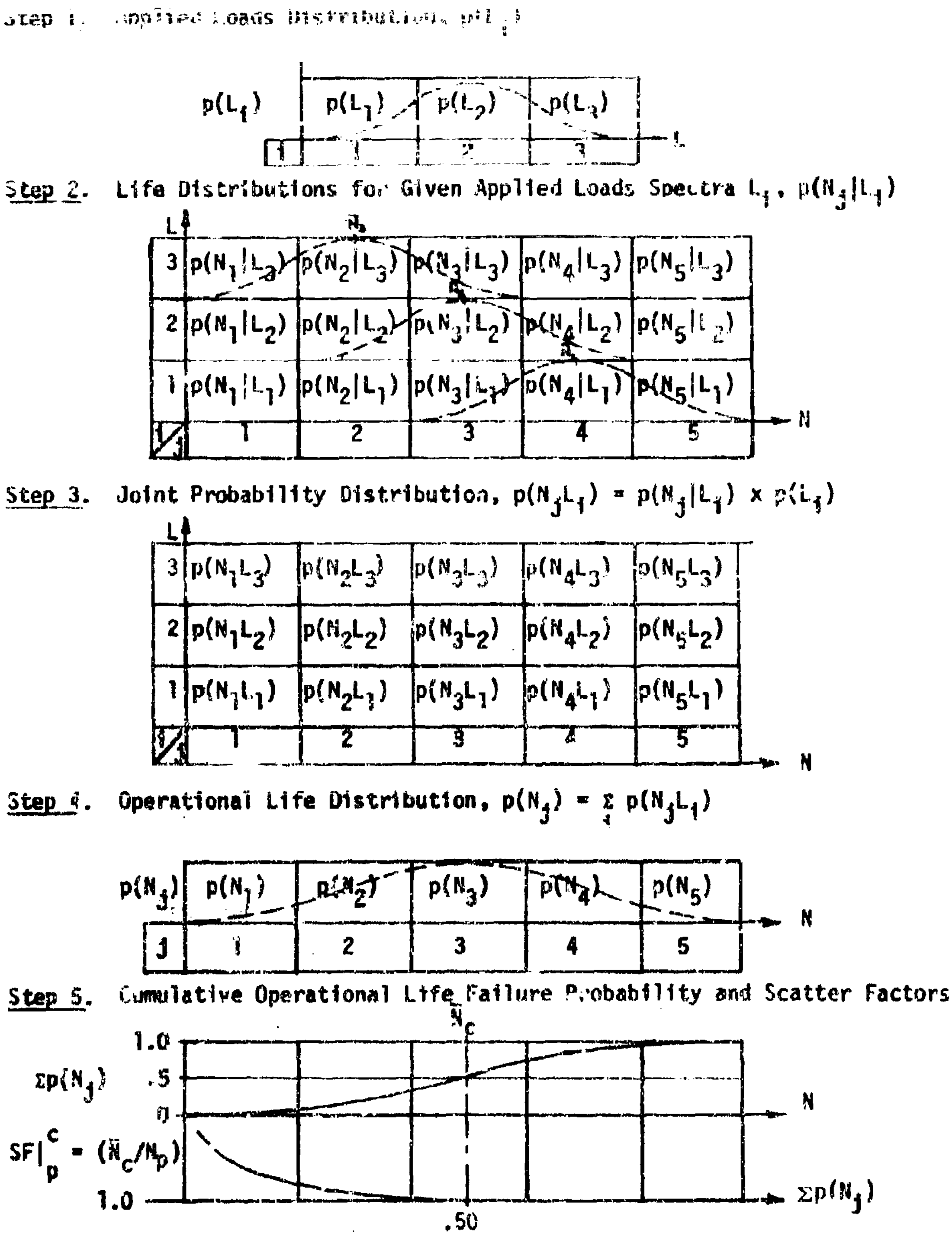


FIGURE 4. THE CONCEPT OF JOINT PROBABILITY DISTRIBUTION AND OPERATIONAL LIFE SCATTER FACTORS

$\frac{L_i-\bar{L}}{\sigma_L}$	$p(L_i)$	i \ j	1	2	3	4	5	6	7	8	9	10	11	12	13
3.5 to 2.5	.0062	7	.0062 / .00004	.0606 / .00038	.2417 / .00150	.383 / .00237	.2417 / .00150	.0606 / .00038	.0062 / .00004	~0	~0	~0	~0	~0	~0
2.5 to 1.5	.0606	6	~0	.0062 / .00038	.0606 / .00367	.2417 / .01465	.383 / .02321	.2417 / .01465	.0606 / .00367	.0062 / .00038	~0	~0	~0	~0	~0
1.5 to .5	.2417	5	~0	~0	.0062 / .00150	.0606 / .01465	.2417 / .05842	.383 / .09257	.2417 / .05842	.0606 / .01465	.0062 / .00150	~0	~0	~0	~0
.5 to -.5 ($\bar{L}$)	.383	4	~0	~0	~0	.0062 / .00237	.0606 / .02321	.2417 / .09257	.383 / .14669	.2417 / .09257	.0606 / .02321	.0062 / .00237	~0	~0	~0
-.5 to -1.5	.2417	3	~0	~0	~0	~0	.0062 / .00150	.0606 / .01465	.2417 / .05842	.383 / .09257	.2417 / .05842	.0606 / .01465	.0062 / .00150	~0	~0
-1.5 to -2.5	.0606	2	~0	~0	~0	~0	~0	.0062 / .00038	.0606 / .00367	.2417 / .01465	.383 / .02321	.2417 / .01465	.0606 / .00367	.0062 / .00038	~0
-2.5 to -3.5	.0062	1	~0	~0	~0	~0	~0	~0	.0062 / .00004	.0606 / .00038	.2417 / .00150	.383 / .00237	.2417 / .00150	.0606 / .00038	.0062 / .00004
		$p(N_j)$	.00004	.00075	.0067	.0340	.1078	.2152	.2710	.2152	.1078	.0340	.0067	.00076	.00004

Column width: $\sigma_{N|L}$ ($\bar{N}_c$ at the boundary between columns 7 and 8)

$m'_p = \left(\frac{\log N_j - \overline{\log N_c}}{\sigma_{N\mid L}}\right)$	-6.5	-5.5	-4.5	-3.5	-2.5	-1.5	-.5	.5	1.5	2.5	3.5	4.5	5.5	6.5
$\Sigma\, p(N_j)$	~0	.00004	.0008	.0075	.0415	.1493	.3643	.6355 (.5)	.8507	.9585	.9925	.9992	.99996	~1.0

$SF|_p^c = \text{Antilog}\,(|m_p|\sigma_N) = \text{Antilog}\,(|m'_p|\sigma_{N|L})$, $\overline{\log N}_{i+1} = \overline{\log N}_i - \sigma_{N|L}$

$= (\bar{N}_c/N_p)$, $N_p < \bar{N}_c$, $\sigma_N = 1.43\,\sigma_{N|L}$

$= (N_p/\bar{N}_c)$, $N_p > \bar{N}_c$, m_p = Number of σ_N from N_c for a given p = I of the Normal distribution.

FIGURE 5. HYPOTHETICAL OPERATIONAL FATIGUE LIFE JOINT PROBABILITY DISTRIBUTION - BIVARIATE NOR...

$\left(\frac{L_i-\bar{L}}{\sigma_L}\right)$	$p(L_i)$	i	$\sigma_p \rightarrow$					$\sigma_{N\|L}$			
3.5 to 2.5	.0062	7	.00245	.00275	.0095	.0402	.1811	.528	.1811	.0402	[illegible]
			.000015	.00002	.00006	.00025	.00112	.00327	.00112	.00025	[illegible]
2.5 to 1.5	.3606	6	.00140	.00105	.00275	.0095	.0402	.1811	.528	.1811	[illegible]
			.000085	.00006	.00017	.00059	.00244	.01098	.03200	.01098	[illegible]
1.5 to .5	.2417	5	.00085	.00055	.00105	.00275	.0095	.0401	.1811	.528	[illegible]
			.000205	.00013	.00025	.00067	.00230	.00972	.04377	.12762	[illegible]
.5 to -.5 ($\bar{L}$)	.383	4	.00053	.00032	.00055	.00105	.00275	.0095	.0402	.1811	[illegible]
			.000203	.00012	.00021	.00040	.00105	.00364	.01540	.06936	[illegible]
-.5 to -1.5	.2417	3	.00034	.00019	.00032	.00055	.00105	.00275	.0095	.0402	[illegible]
			.000082	.00005	.00008	[illegible]	.00025	.00067	.00230	.00972	[illegible]
-1.5 to -2.5	.0603	2	.00022	.00012	.00019	.00032	.00055	.00105	.00275	.0095	[illegible]
			.000013	.00001	.00001	.00002	.00003	.00006	[illegible]	.00058	[illegible]
-2.5 to -3.5	.0062	1	.00014	.00008	.00012	.00019	.00032	.00055	.00105	.00275	[illegible]
			.000001	.00000	.00000	.00000	.00000	.00003	.00001	.00002	[illegible]
	i / j			1	2	3	4	5	6	7	8
	$p(N_j)$			.00039	.00078	.00236	.00719	.02834	.0947	.21513	[illegible]
$m_p' = \left(\frac{\log N_j - \overline{\log N_c}}{\sigma_{N\|L}}\right)$			-7.5	-6.5	-5.5	-4.5	-3.5	-2.5	-.5	[illegible]	[illegible]
	$\Sigma p(N_j)$		.00060	.00090	.00177	.00382	[illegible]	.03535	[illegible]	[illegible]	[illegible]

$SF|_p^c = \text{Antilog}(m_p' \sigma_{N|L}) = (\bar{N}_c/N_p)$; $N_p < \bar{N}_c$; $\sigma_{N|L} = .13$

$= (N_p/\bar{N}_c)$; $N_p > \bar{N}_c$; $\overline{\log N_{j+1}} = \overline{\log N_j} + \sigma_p$

FIGURE 6. HYPOTHETICAL OPERATIONAL FATIGUE LIFE JOINT PROBABILITY DISTRIBUTION - p(L) NORMAL, p(N|L) TEST DATA DISTRIBUTION

L_i	$p(L_i)$	i							
V	.069	4	.0086	.1500	.5900	.8600	.9650	.9910	$\Sigma p(N_j \mid L_i)$
			.0006	.0104	.0407	.0593	.0666	.0684	$\Sigma p(N_j, L_i)$
I	.366	3	.0000	.0028	.0670	.2600	.5400	.7500	
			.0000	.0010	.0245	.0952	.1976	.2745	
II-III	.459	2	.0000	.0001	.0350	.1300	.4100	.6500	
			.0000	.0000	.0161	.0826	.1882	.2384	
IV	.106	1	.0000	.0000	.0160	.1100	.2900	.5000	
			.0000	.0000	.0017	.0117	.0307	.0530	
		j	1	2	3	4	5	6	
		$\Sigma p(N_j)$	.0006	.0114	.0830	.2488	.4834	.6943	
N_j = Flt. Hrs. =			6,500	10,000	15,000	20,000	25,000	30,000	

$\sigma_{N|L} = 0.14$

For L_i, $p(L_i)$ and $\bar{N}_i$ Ref. Table 4

$\bar{N}_c$ = 25,400 Flt. Hrs., Ref. Fig. 10

$SF|_p^c = (\bar{N}_c/N_p)$, Ref. Fig. 9

FIGURE 7. ACTUAL TRANSPORT AIRCRAFT OPERATIONAL FATIGUE LIFE JOINT PROBABILITY DISTRIBUTION - $p(N|L)$ NORMAL DISTRIBUTION

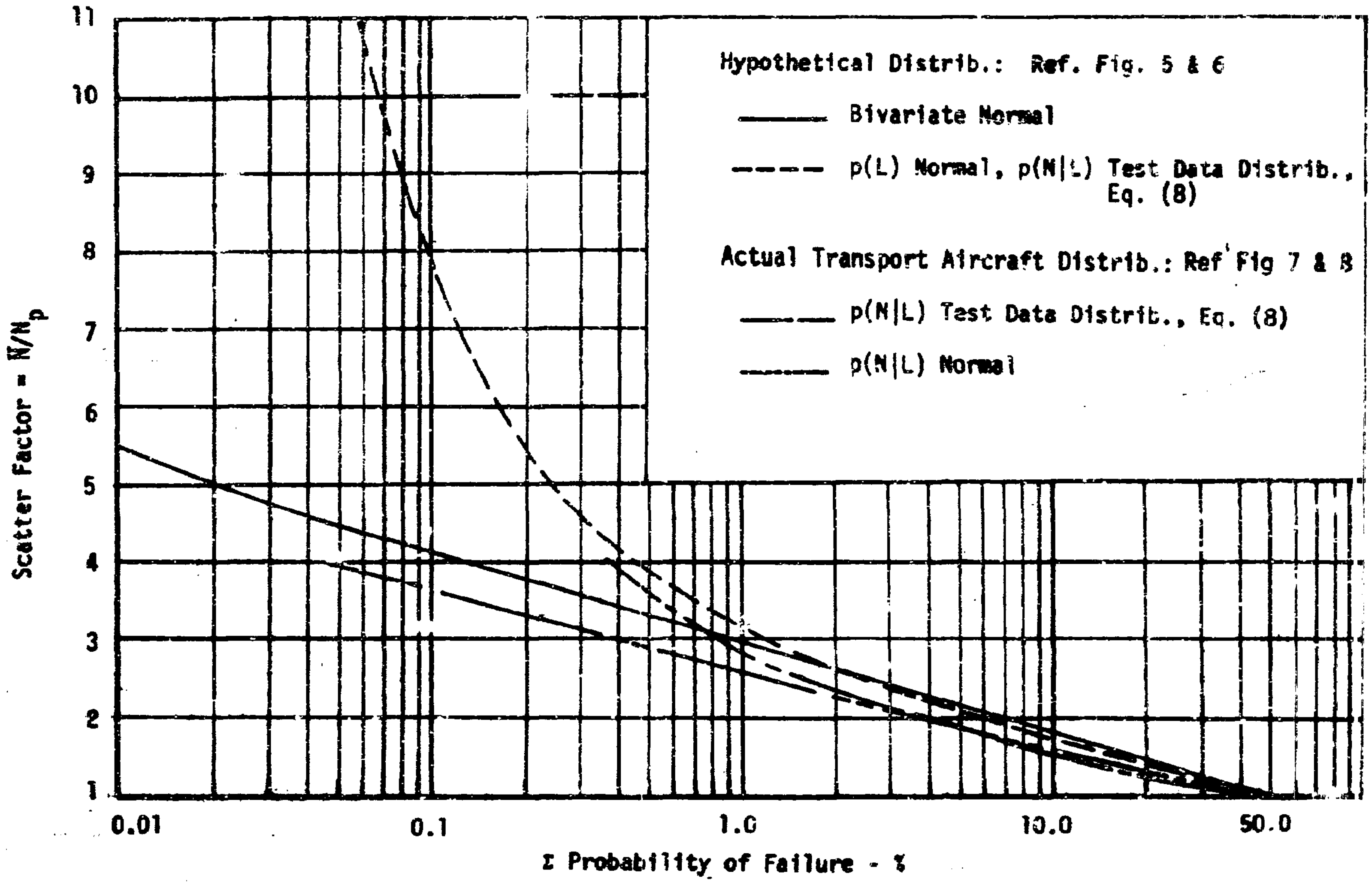


FIGURE 9. OPERATIONAL FATIGUE LIFE SCATTER FACTORS BASED ON JOINT PROBABILITY DISTRIBUTION CONCEPT

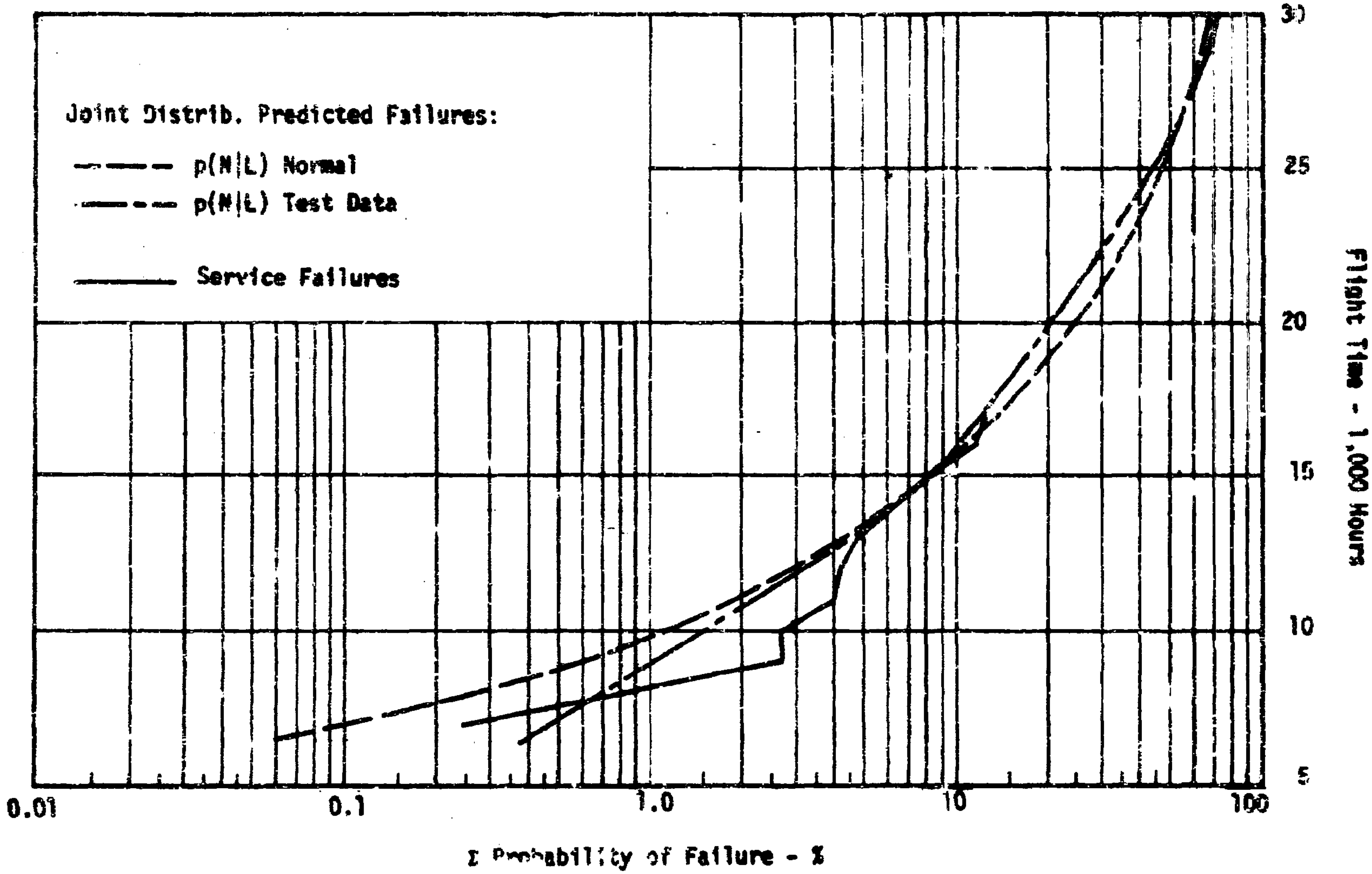


FIGURE 10. COMPARISON OF PREDICTED AND ACTUAL TRANSPORT AIRCRAFT STRUCTURAL ELEMENT PROBABILITY OF FAILURE DISTRIBUTION

FATIGUE DAMAGE RATES AND DESIGN CHARTS

Fatigue analysis of aircraft structures is a complex and time consuming process. The fatigue damage calculation-life prediction computer program, presented in Part XI of this report, Reference 8, is an efficient engineering tool for the execution of detail large scale fatigue analysis problems. However, even the computer program is sometimes a tedious procedure when quick approximate life predictions, such as in the early design stages, must be obtained. This section presents one possible approach and examples for the development of general fatigue strength design-damage rate charts for rapid estimation of structural fatigue lives. The computer program of Reference 8 is an extremely useful tool in the development of such charts and was utilized throughout this study. Linear cumulative damage rule was used for all damage calculations.

1. Generalized Loads Spectrum Formats

Aircraft fatigue incremental loads spectra usually can be represented in the following equation forms:

Exponential Distribution,

$$\Sigma n_j = \sum_i N_{0_i} e^{-\Delta y_j / b_i} \quad , \quad \begin{matrix} i = 1,2,3 \ldots \\ \Delta y = 0 \ldots . \Delta y' \end{matrix} \tag{35}$$

or

Normal Distribution,

$$\Sigma n_j = \sum_i N_{0_i} e^{-\Delta y_j^2 / 2\sigma_i^2} \quad , \quad \begin{matrix} i = 1,2,3 \ldots \\ \Delta y = 0 \ldots . \Delta y' \end{matrix} \tag{36}$$

where,

Δy = incremental load factor, bending moment, load, stress, etc. (to be called 'load' for general discussion).

$\Delta y'$ = largest incremental load in a spectrum.

Σn_j = frequency of occurrence of the incremental loads $\Delta y \geq \Delta y_j$; cumulative cycles.

N_0 = frequency of occurrence of all loads $\Delta y > 0$; cycles per time, distance, number of flights, etc.

b, σ = spectrum magnitude parameter in units of Δy.

The summation sign on the right side of equations (35) and (36) implies that as many terms as are needed can be used to define the spectrum accurately. Description of graphical approximations of a given spectrum by these equations is presented in Part II of this report, pages 11 to 14, Reference 8.

[illegible]
must be specified

b. Spectrum Constant Y Load

The cyclic loads for a spectrum with Δy variable and Y constant can take three different forms in terms of Δy and Y, depending whether Y is the constant mean, maximum or minimum spectrum load:

Y	CYCLIC LOAD	Δy	Y_{max}	Y_{min}	
Y_m	$(Y \pm \Delta y)$	Y_a	$(Y + \Delta y)$	$(Y - \Delta y)$	(37)
Y_{max}	$(Y - \frac{1}{2}\Delta y) \pm \frac{1}{2}\Delta y$	$Y_r = 2Y_a$	Y	$(Y - \Delta y)$	(38)
Y_{min}	$(Y + \frac{1}{2}\Delta y) \pm \frac{1}{2}\Delta y$	$Y_r = 2Y_a$	$(Y + \Delta y)$	Y	(39)

where, Y_m = mean load $= (Y_{max} + Y_{min})/2$

Y_a = load amplitude $= (Y_{max} - Y_{min})/2$

Y_{max} = maximum load

Y_r = load range $= 2Y_a = Y_{max} - Y_{min}$

Y_{min} = minimum load

Thus, a given spectrum, with one of the cyclic load parameters Y_m, Y_{max}, or $Y_{min} = Y$ = constant, can be completely defined in terms of N_o, b (or σ), $\Delta y'$, and Y by equations (35) to (39).

2. Damage Rate Charts

Fatigue damage rate of a structural element is a function of the applied loads spectrum and the element fatigue strength quality. If an average K_f value, the empirical fatigue stress concentration factor, can be considered to be a measure of the fatigue strength quality, and the applied loads spectrum is defined by the parameters described in the preceding paragraph, then the damage rate of one term of equation (35) or (36) can be completely defined as,

$$(D/N_\gamma) = f[b \text{ (or } \sigma), \Delta y', Y, K_f] \tag{40}$$

and also a function of the cyclic loads format, equations (37) to (39).

To illustrate the development and to present samples of damage rate charts, damage rate calculations were performed for a range of K_f values as represented by 7075-T6 aluminum bare sheet S-N data. The S-N data and the corresponding K_f values were taken from Reference 9. A total of six K_f values were considered, ranging from 1.37 to 3.64. The corresponding range of K_t values is from 1.5 to 5.0. The six S-N diagrams, as used in the damage rate calculations, are presented by Figures 11, 12, and 13. The stresses are specimen net area stresses. Figure 14 presents a family of damage rate curves for $K_f = 2.62$ and the spectrum and cyclic loads in the form of equations (35) and (37); the symbol Y is replaced by S, for stress, psi. The damage rate curves encompass a range of b, $\Delta S' = S_a'$, and $S = S_m$ values representative of typical aircraft fatigue loads spectra. For a given material, a complete set of damage rate curves would encompass a range of K_f values representative of aircraft structure fatigue quality as well as the other spectrum and cyclic loads formats, equations (36), (38), and (39). Samples of damage rate curves for a range of K_f values and the other spectrum and cyclic loads formats are shown in Figures 15 and 16. Attempts to normalize a family of damage rate curves into a single general graph were not successful. However, one other form of presenting fatigue strength allowables under spectrum loading is illustrated by Figure 17. For a given K_f, spectrum, and cyclic load format, the damage rates, for one value of b or σ, can be converted into a constant life diagram where the allowable life, N_s, under spectrum loading is the inverse of the damage rate D/N_o. Figure 17 presents the constant life curves of the $K_f = 2.62$ damage rates shown in Figure 14 for b = 15,000 psi. The prime with any cyclic load parameter indicates the value associated with the largest incremental load, $\Delta y' = \Delta S'$, in the spectrum.

Use of the damage rate charts may be best illustrated by several examples. First, let us assume that the damage rates are based on statistically established S-N data where the S-N curves represent mean values with an associated confidence level. Thus, the calculated life under spectrum loading will be the mean life with the confidence level of the S-N data.

Example 1. For a structural element with fatigue quality of $K_f = 2.62$, find the mean life if the stress spectrum for 30,000 flight hours is represented by $\Sigma n = \Sigma N_{oi} e^{-\Delta s/b_i}$, i = 1,2 , and the cyclic loads are $S_m \pm \Delta S$, where:

i	N_{oi}	b_i ~ psi	S_m ~ psi	$\Delta S' = S_a'$ ~ psi
1	10^4	7,500	10,000	20,000
2	3×10^5	2,500	10,000	20,000

The damage rates for the two terms are obtained from Figure 14 and the total damage for 30,000 flight hours is:

i	$D/(N_o = 10^5)$	D/N_{oi}
1	1.13	.113
2	.069	.207
		.320

The predicted mean life is (30,000/.32) = 93,800 flight hours.

Example 2. Taking the problem of Example 1, consider that the aircraft utilization has changed in such a manner that the stress spectrum for 30,000 flight hours becomes:

i	N_{0_i}	$b_t \sim psi$	$S_m \sim psi$	$S_a' \sim psi$
1	9×10^3	7,500	10,000	20,000
2	2.7×10^5	2,500	10,000	20,000
3	4×10^3	7,500	15,000	25,000

Again, the damage rates are obtained from Figure 14 and the total damage for 30,000 flight hours is:

i	$D/(N_0 = 10^5)$	D/N_{0_i}
1	1.13	.102
2	.069	.186
2	4.3	.172
		.460

The predicted mean life is = 30,000/.46 = 65,200 flight hours.

The above examples, although for hypothetical spectra, illustrate the rapidity of predicting fatigue lives from damage rate charts, such as those of Figure 14. Of course, in real problems the spectrum parameters will not always correspond to the values of the damage rate curves presented and a certain amount of crossplotting of the data will be necessary.

Several aspects of the damage rate concept which require further attention are the fatigue quality estimation of the structural element and the availability of statistically reliable S-N data and the validity of the linear damage rule. At present, analytical methods are not available to calculate the fatigue quality of a complex structural element, whether it is measured in terms of K_t or K_f. The quality must be estimated by testing the element or by comparing to a similar element with a known fatigue quality.

3. Ground-Air-Ground Cycle Damage Rates

Most aircraft structural elements, due to the combination and sequence of the environmental loadings during a flight, experience a significant cyclic loading called the ground-air-ground (GAG) cycle. Reference 10 presents a detailed discussion of the GAG cycle concept. The GAG cycle is defined for each individual flight by the maximum and minimum loads which occur during that flight, including the ground loads. For a large number of flights, the GAG cycles will define a spectrum type loading because each flight, theoretically, will experience a different GAG cycle. Such spectrum generally will not have a constant mean, maximum or minimum load. Consequently, damage rates for spectra which exhibit this property, such as those presented in this section, are usually not applicable to the GAG cycle spectrum.

A typical transport aircraft structural element GAG cycle spectrum is shown by Figure 18. It is seen that neither the mean, maximum nor minimum cyclic load is constant for the GAG cycle spectrum. The damage rate of this spectrum cannot be defined by the parameters of equation (40). However, the ground and flight loads spectra can be individually defined in this form by equations (35) and (37). The fatigue damage calculation computer program of Reference 8 has the capability of calculating the GAG cycle spectrum damage rate, given the above definition of the ground and flight loads spectra and the number of flights (or landings), f_{GAG}, represented by the spectra. Thus, symbolically, GAG cycle spectrum damage rate can be defined as a function,

$$(D/f_{GAG}) = f[(N_o,b,\Delta y',Y)_{Ground}, (N_o,b,\Delta y',Y)_{Flight}, f_{GAG}, K_f] \qquad (41)$$

To develop a family of damage rate curves to encompass a complete matrix of the above parameters would be almost an insurmountable task. Figure 19 presents samples of GAG cycle spectrum damage rates when all parameters of equation (41), except two, are held constant. The ground and flight loads spectra, over the GAG cycle spectrum loads range, are represented by one term of equation (35).

On the basis of the GAG cycle spectrum damage rate calculations in this study, the following approximate and simple procedure for the estimation of the GAG cycle spectrum damage rate is recommended: calculate the damage rate corresponding to the GAG cycle spectrum maximum and minimum loads which are exceeded in 40 percent of the flights. Following this procedure, the damage rate per 1,000 flights of the Figure 18 GAG cycle spectrum would be calculated as 1000/N, where, N, cycles to failure would be obtained from S-N data for cyclic loading, $S_{max} = 15{,}100$ and $S_{min} = -7{,}100$. These stress values in Figure 18 correspond to the GAG cycle spectrum loads at $\Sigma n = 400$.

A common uncertainty exists about the effect of the GAG cycles on fatigue life of fighter type aircraft (high design load factors, low 1.0g stresses, maneuver loads critical) as compared to transport type aircraft (low design load factors, high 1.0g stresses). This uncertainty probably stems from the fact that very little testing has been performed with realistic maneuver plus GAG cycle loadings representative of fighter aircraft as compared to gust plus GAG cycle loadings representative of transport aircraft, see Tables 11 to 14. However, Reference 18 contains fighter type maneuver-GAG cycle loading test data which indicates a similar detrimental affect of the GAG cycles on fatigue life as for transport type aircraft. Consequently, the definition of the GAG cycles, as described in the preceding paragraphs, is considered to be applicable to structural elements of all types of aircraft.

4. Design Charts

For all practical purposes, a complete set of damage rate charts, as previously defined in this section, constitute a basic and completely general set of fatigue strength design charts. Such charts are most useful in the early design stage parametric studies when most of the design parameters have not been finalized. However, in the later stages of design when the aircraft utilization and the applied loads spectra in terms of load factors can be firmly established, the fatigue strength of the structural element becomes a function of the fatigue quality of the element and the operational stress levels. Figure 20 presents such design charts for the applied loads spectrum of Figure 18 and the fatigue quality as defined for 7075-T6 aluminum sheet by the S-N data, Figures 11 to 13. The design charts were developed with the aid of the damage rates established for the above S-N data in this study. The applied loads spectrum, per 1,000 flights, was defined in terms of load factors in the following form:

Ground Loads - Taxi:

$\Sigma n = N_o e^{-\Delta g/b}$, $N_o = 2.5 \times 10^6$ cycles

$\Delta g' = .8$, largest incremental load factor

$b = .048$

Load Cycle = $1 \pm \Delta g$

Flight Loads - Maneuver and Gust:

$\Sigma n = N_{o_i} e^{-\Delta g/b_i}$,

i =	1	2
N_{o_i} =	7.5×10^2	2×10^5
b_i =	.224	.082
$\Delta g'$ =	2	2

Load Cycle = $1 \pm \Delta g$

GAG Cycle:

$$S_{max} = f(1 + \Delta g)_{flight} = f(\text{Flight LF} = 1.51)$$

$$S_{min} = f(1 + \Delta g)_{ground} = f(\text{Ground LF} = 1.42)$$

where load factors (LF) are taken from Figure 18 at $\Sigma n = 400$.

A linear relationship was considered between load factors and stress, i.e., $\Delta S = S_m (\Delta g)$ and $S_{max}, S_{min} = (1 \pm \Delta g)\, S_m$. The ground and flight loads mean stresses were related as $S_{m_G} = -(S_{m_F}/2)$. The stresses are net area values.

Figure 20 presents the fatigue strength allowables for any 7075-T6 aluminum structural element for the applied loads spectrum of Figure 18. The use of such charts for design purposes may be best illustrated by an example:

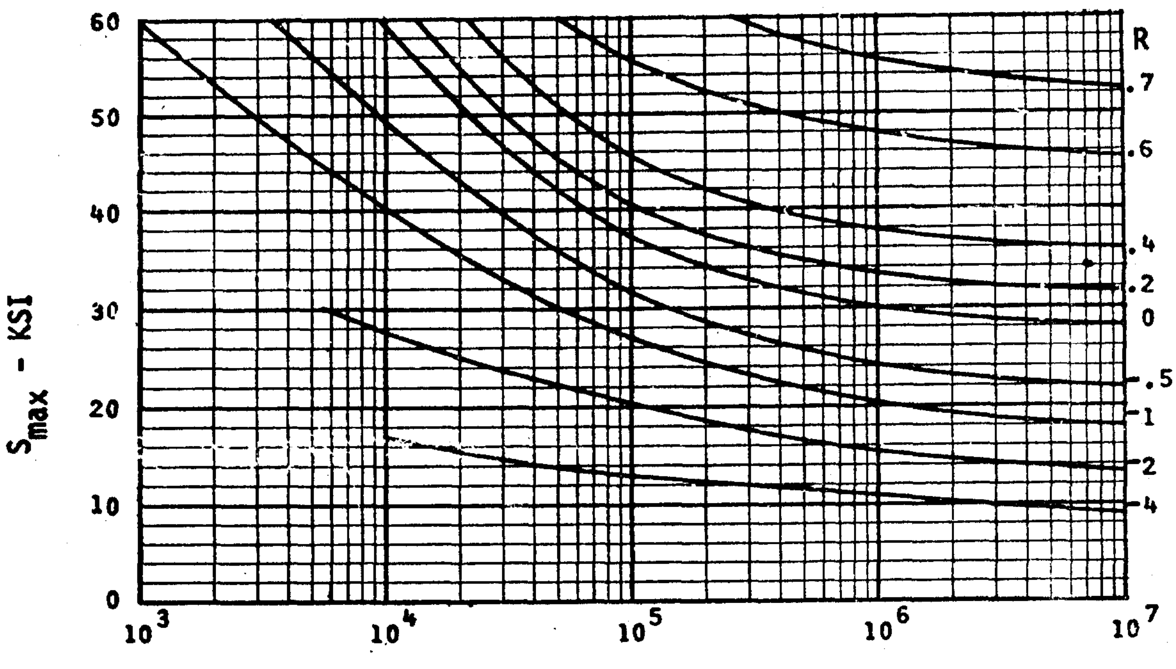
Problem: Design a structural element, for the applied loads spectrum of Figure 18, for a life of 50,000 flights with $p \leq 1\%$ probability of failure. The flight one g static strength design net stress is 19,000 psi. The average operating flight one g stresses are 80% of the design values, 19,000(.8) ≈ 15,000 psi.

Solution: First, from Section II, consider a scatter factor of 3.0 for $p \leq 1\%$. Therefore, the element must be designed for a mean life of 50,000(3) = 150,000 flights. The life and static strength requirements are satisfied by any combination of $K_f \leq 1.8$ and $S_{mF} \leq 15{,}000$ psi as illustrated in Figure 20. The optimum design, with respect to structural weight can be attained at $S_{mF} = 15{,}000$ psi if the structural element fatigue quality is $K_f \leq 1.8$, where $K_f = 1.8$ correspond to $\bar{N} = 150{,}000$ and $S_{mF} = 15{,}000$. If the fatigue quality is $K_f > 1.8$, then the critical strength design condition is fatigue strength and the design one g stress will be less than $15{,}000/.8 \approx 19{,}000$. For example, if $K_f = 2$, $S_{mF} =$ 13,500 and the design one g net stress becomes $13{,}500/.8 \approx 17{,}000$. Similarly, for $K_f = 2.74$, the design one g net stress is $10{,}000/.8 = 12{,}500$.

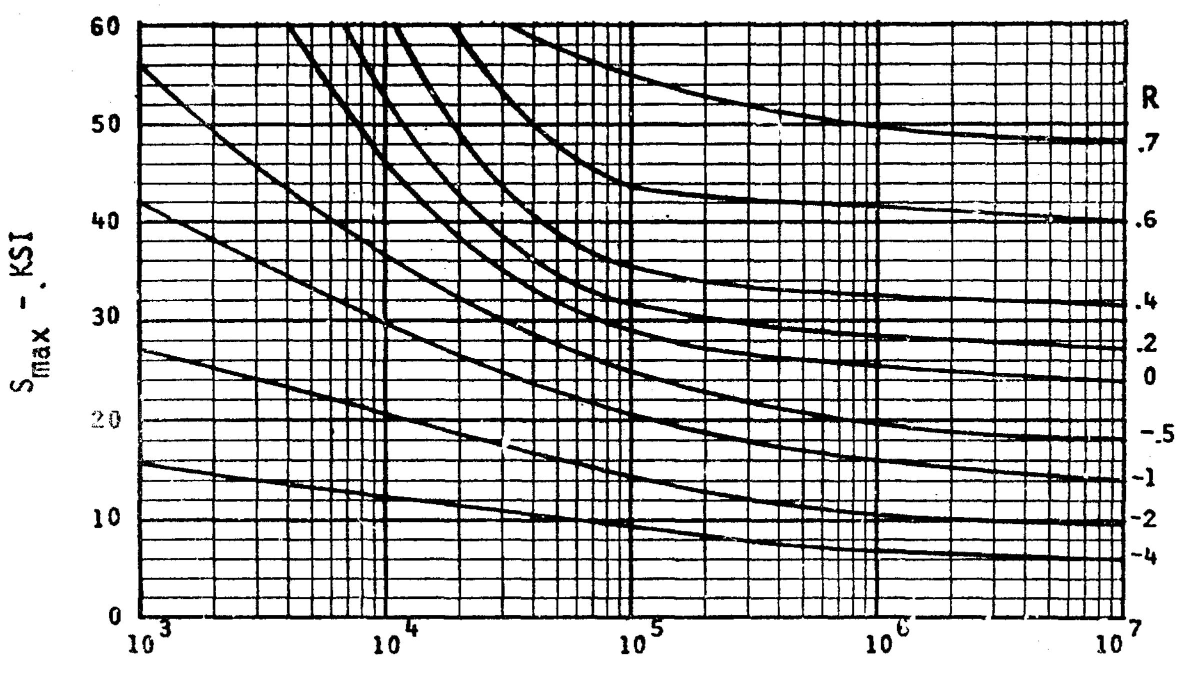
5. Concluding Remarks

Development of fatigue strength design charts for a given loads spectrum in terms of the fatigue quality of the structural element, K_f, and design 1.0g stresses, as illustrated by Figure 20, appears to be a possible and practical approach. Also, development of completely generalized fatigue damage rate charts, to encompass all loads spectrum parameters is possible with the exception of the damage rates of the GAG cycle spectrum. GAG cycle damage rates are a function of the composite loads spectrum and involve separate loads spectra parameters. GAG cycle damage rate may be simply approximated by considering the loads which are exceeded in 40% of the flights, or, for more accurate damage rates, directly calculated from the composite spectrum on the basis of the complete GAG cycle spectrum of highest and lowest peak loads.

The linear cumulative damage theory has been used for fatigue life prediction throughout this study. The accuracy of the linear damage rule is often questioned. The most common arguments are: linear damage rule does not account for the loads sequence nor stress interaction. To answer the first argument, operational loads are random and their exact sequence is not known. Thus, testing to an unorthodox sequence of loadings, not representative of service random loads, does not invalidate the use of linear damage rule in aircraft fatigue life prediction. However, because of the stress interaction effects on fatigue life, the true accuracy of the linear damage rule can be checked by testing to random loading spectra which reflect the frequency and magnitude of operational loads. If the spectra were defined in terms of generalized spectra parameters, the results of such tests can be presented and used in the manner described for the damage rate curves, or directly, as spectrum loading S-N curves.

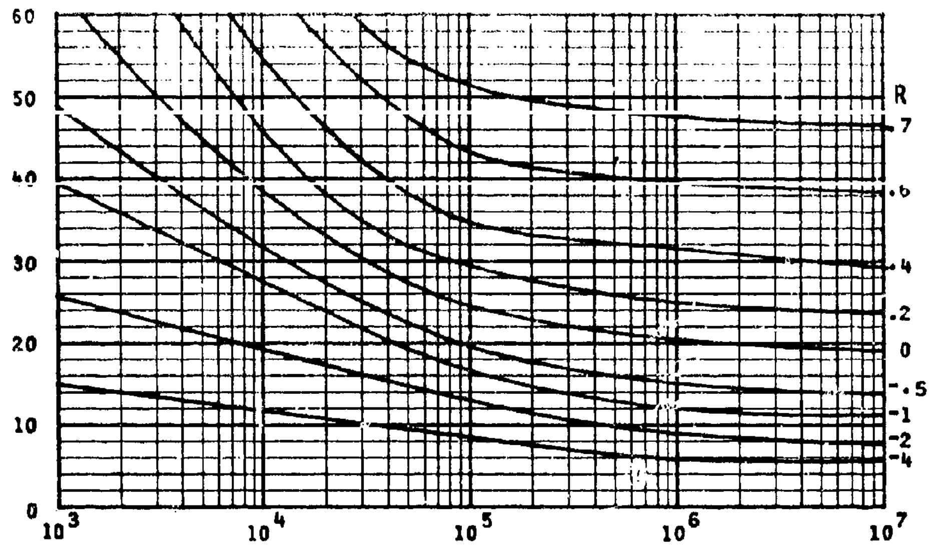


K_t = 1.5, Edge Notch, r = .76 in.; K_f = 1.37

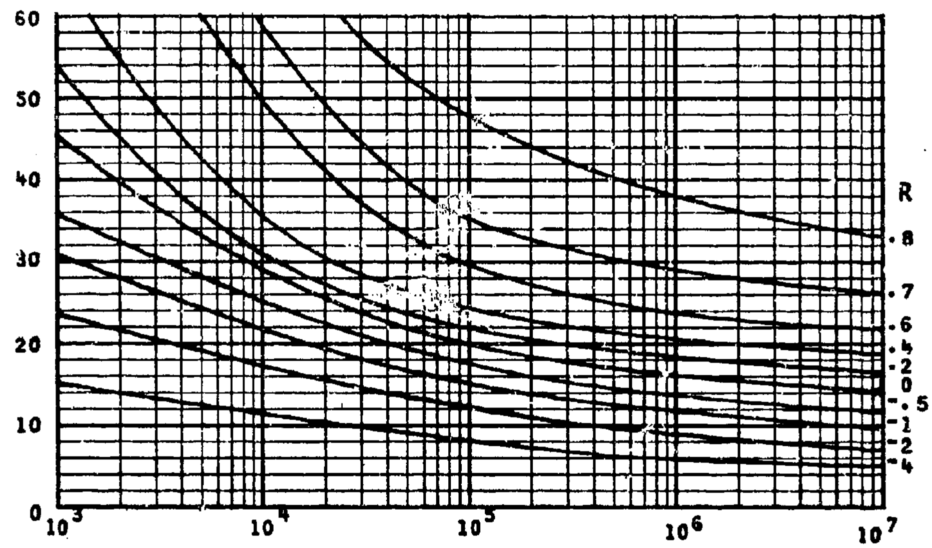


K_t = 2.0, Edge Notch, r = .3175 in.; K_f =1.75

FIGURE 11. S-N DATA: 7075-T6 SHEET AXIAL LOADING, K_t = 1.5 & 2.0

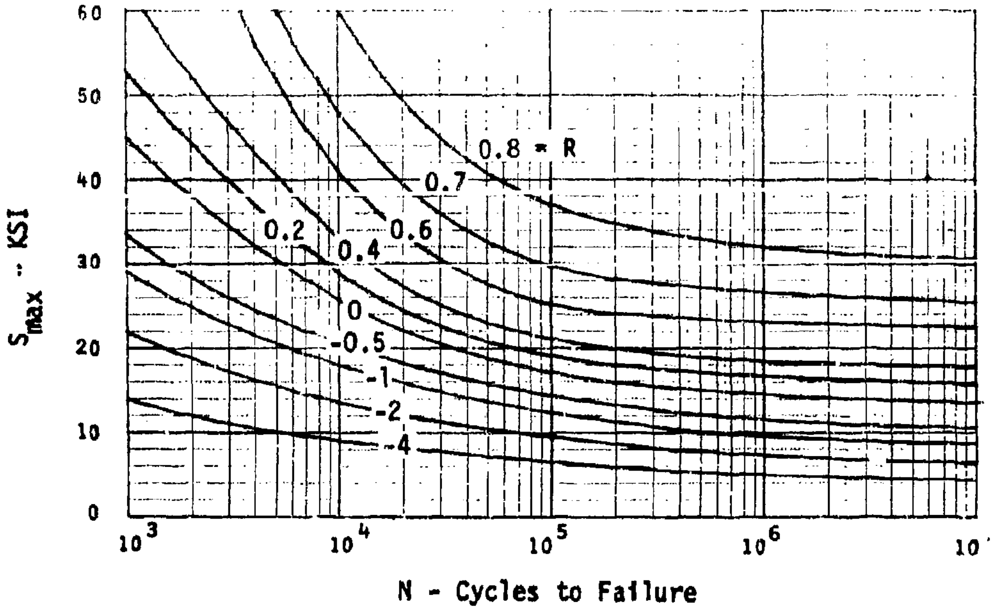


K_t = 2.9, Hole Notch, r = .0313 in.; K_f = 2.07

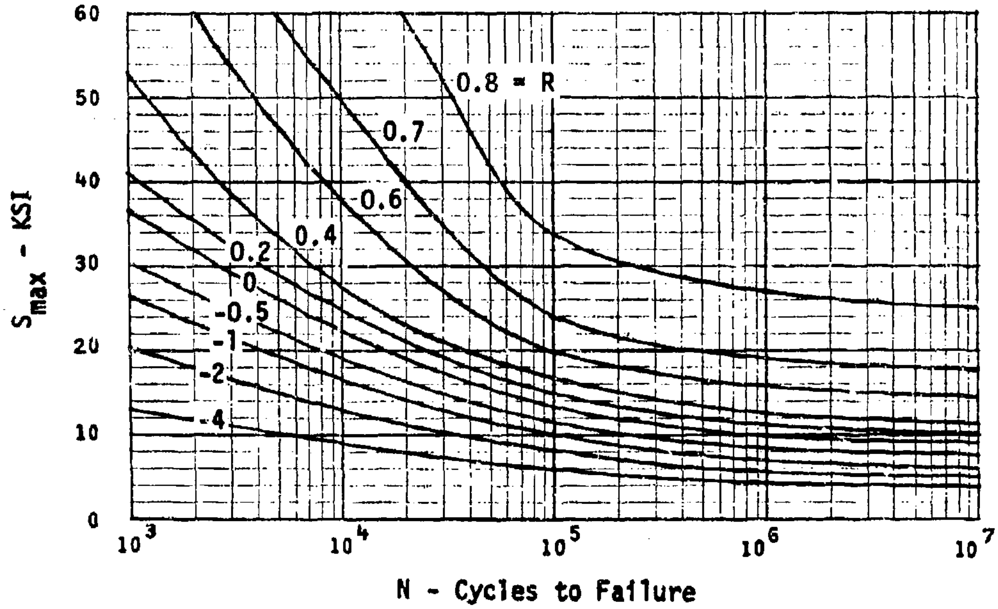


K_t = 4.0, Fillet Notch, r = .0195 in.; K_f = 2.62

FIGURE 12. S-N DATA: 7075-T6 SHEET AXIAL LOADING, K_t = 2.9 & 4.0



K_t = 4.0, Edge Notch, r = .057 in.; K_f = 3.02



K_t = 5.0, Edge Notch, r = .031 in.; K_f = 3.64

FIGURE 13. S-N DATA: 7075-T6 SHEET AXIAL LOADING, K_t = 4.0 & 5.0

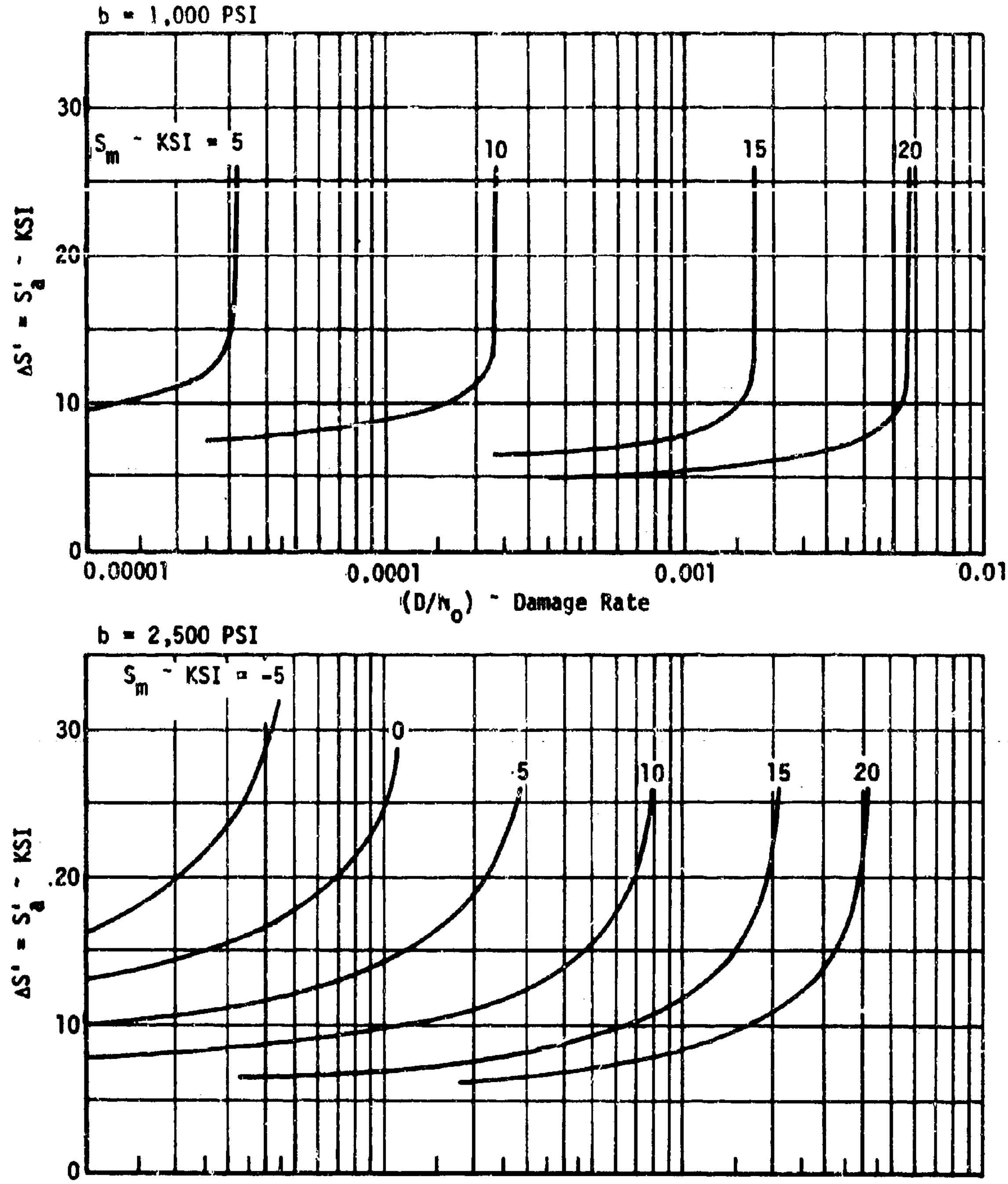


FIGURE 14. 7075-T6 ALUMINUM, K_f = 2.62, DAMAGE RATES FOR $\Sigma n = N_o e^{-\Delta s/b}$ LOADS SPECTRA WITH CONSTANT S_m; $N_o = 10^5$

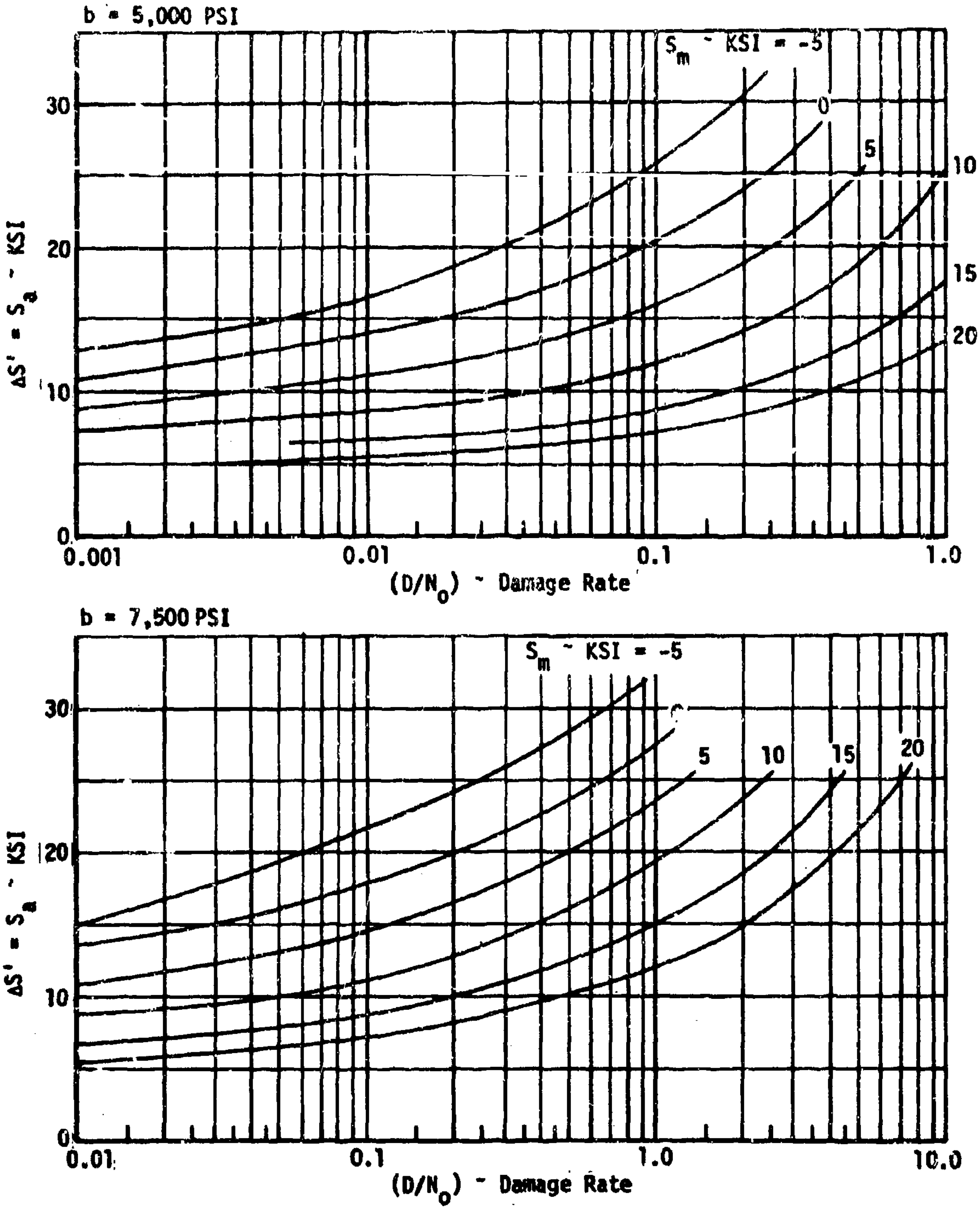
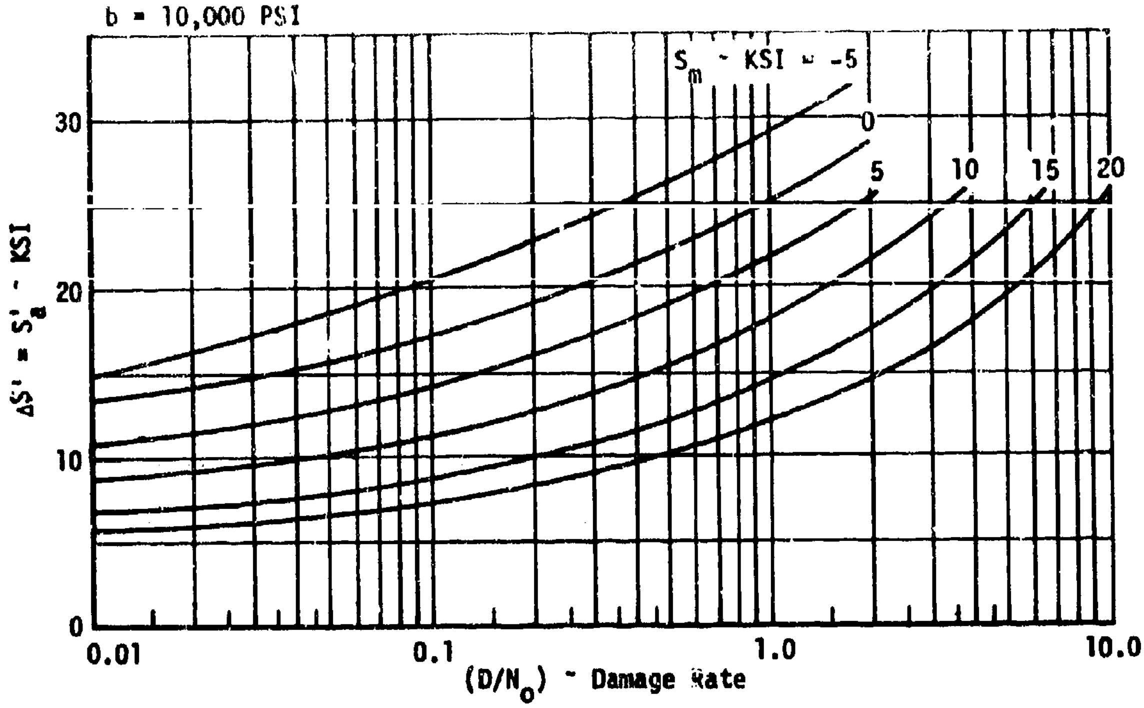


FIGURE 14. 7075-T6 ALUMINUM, K_f = 2.62, DAMAGE RATES FOR $\Sigma\, n = N_o e^{-\Delta s/b}$ LOADS SPECTRA WITH CONSTANT S_m; $N_o = 10^5$ (Continued)



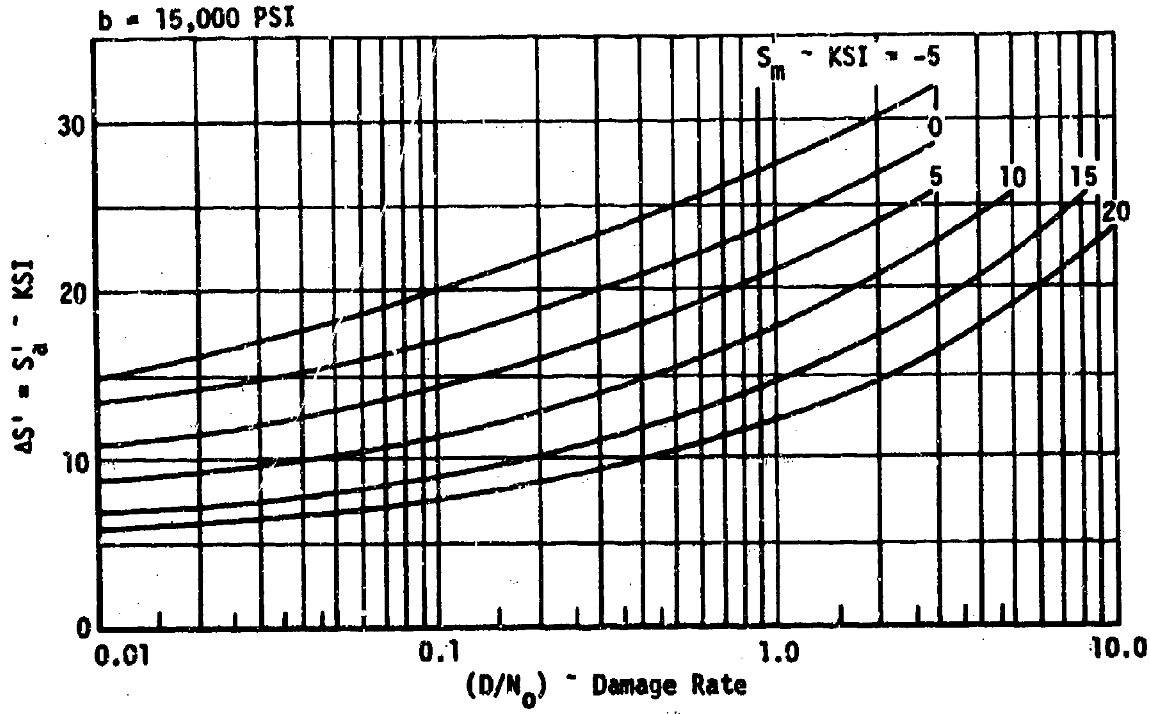


FIGURE 14. 7075-T6 ALUMINUM, $K_f = 2.62$, DAMAGE RATES FOR $\Sigma n = N_o e^{-\Delta s/b}$ LOADS SPECTRA WITH CONSTANT S_m; $N_o = 10^5$ (Concluded)

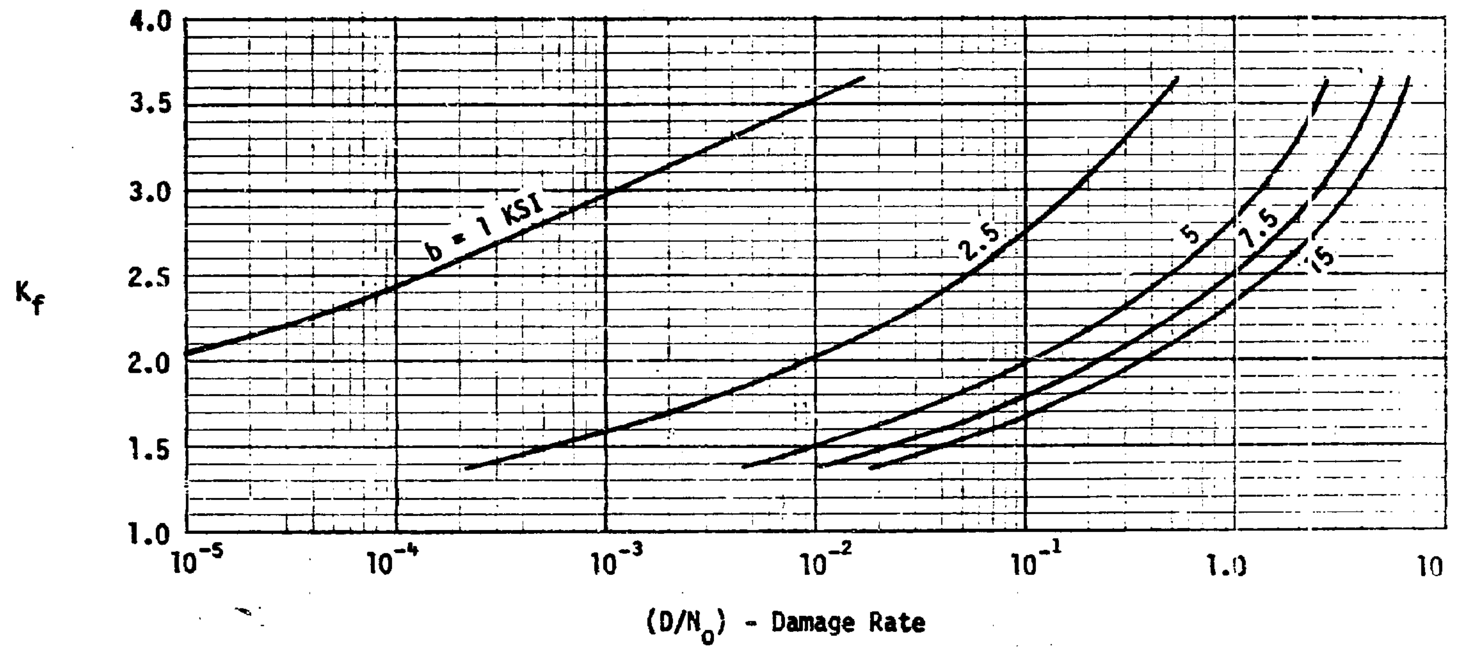


FIGURE 15. 7075-T6 SHEET, K_f = 1.37 to 3.64, DAMAGE RATES FOR $\Sigma n = N_o e^{-\Delta s/b}$ LOADS SPECTRA WITH CONSTANT S_m; $N_o = 10^5$, $S_m = 10{,}000$, $S_a' = 20{,}500$

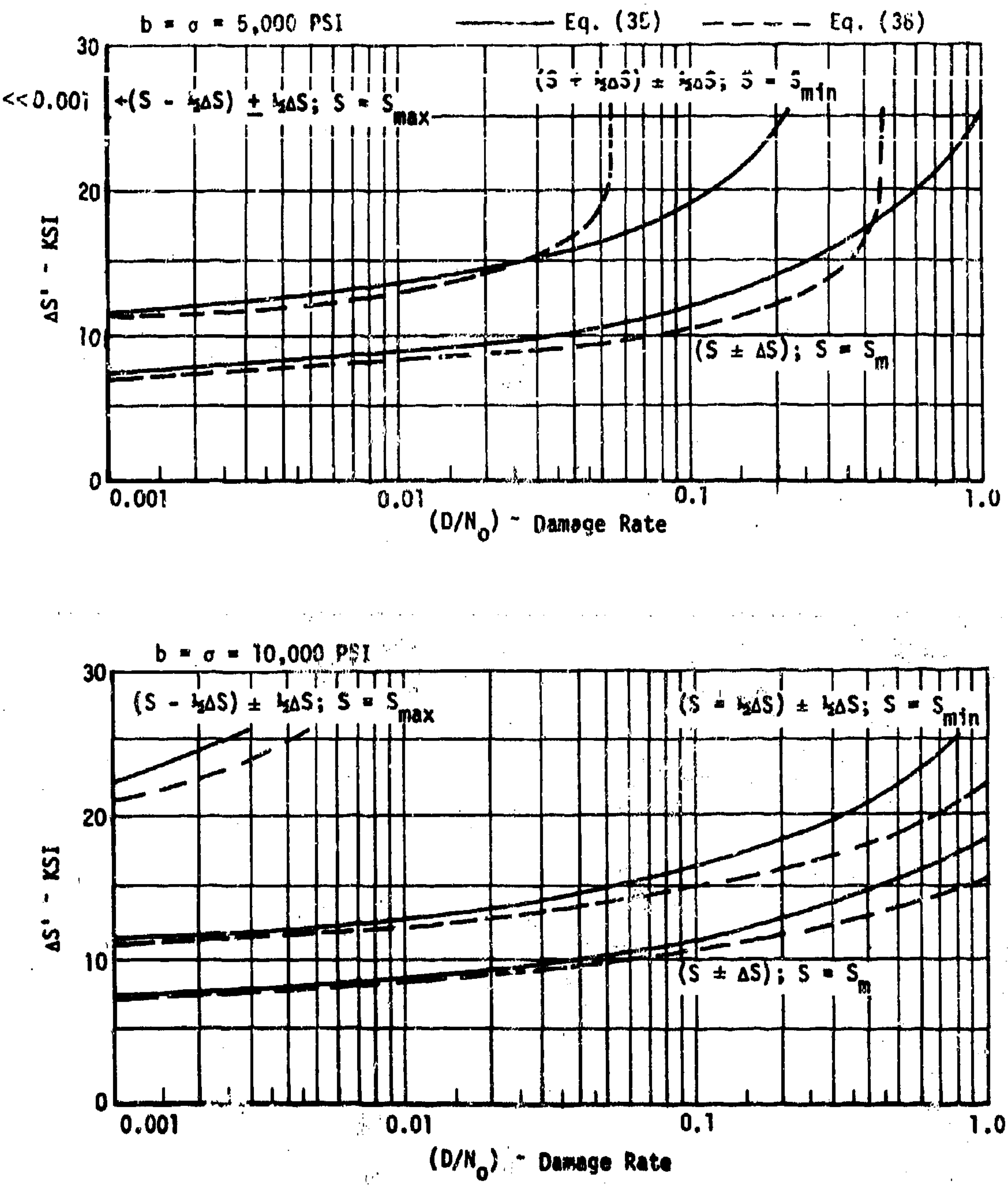


FIGURE 16. COMPARISON OF DAMAGE RATES FOR DIFFERENT SPECTRA AND CYCLIC LOAD FORMATS; 7075-T6 ALUMINUM, $K_f = 2.62$; $N_o = 10^5$, S = 10,000

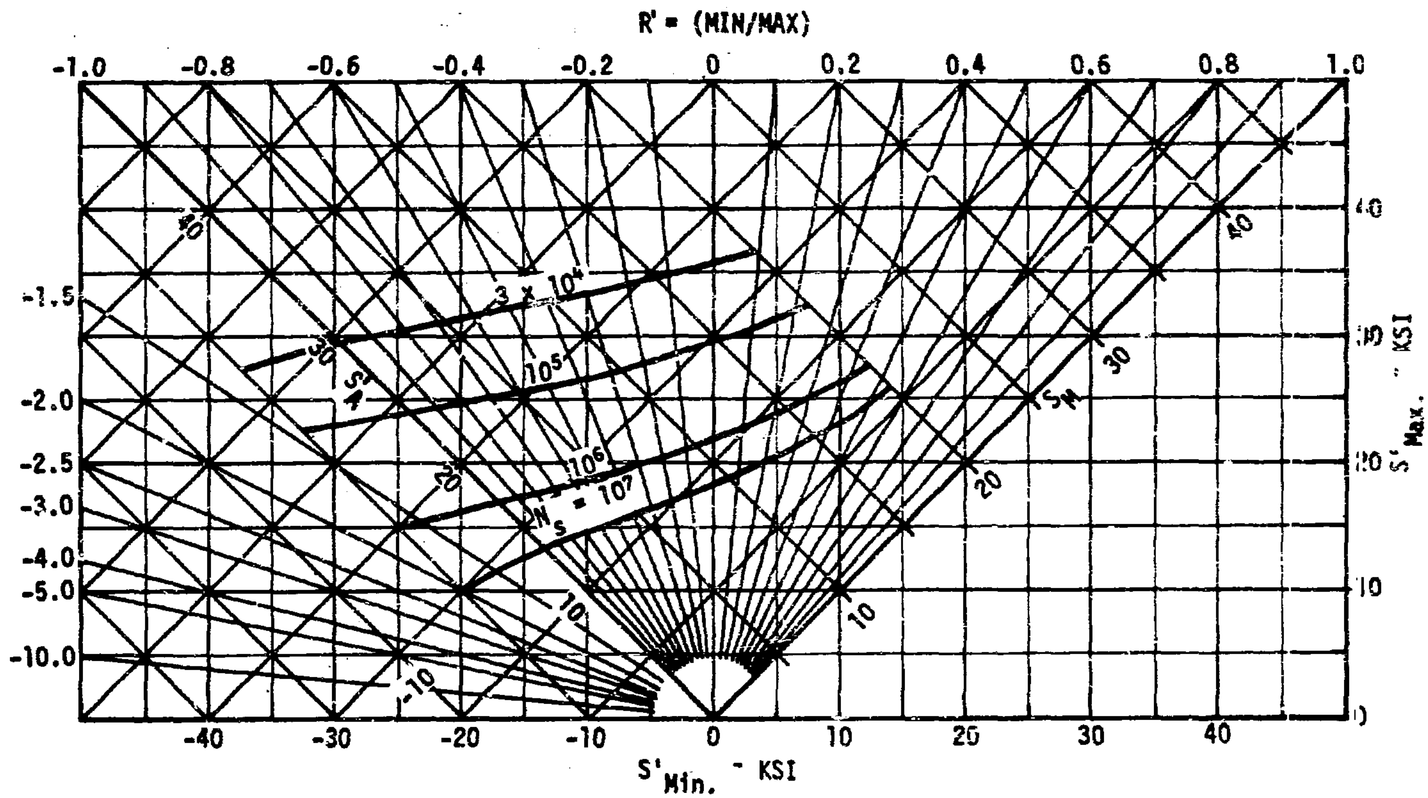


FIGURE 17. FATIGUE CONSTANT LIFE DIAGRAM FOR SPECTRUM LOADINGS OF THE FORM $\Sigma n = N_o e^{-\Delta s/b}$, b = 15,000 psi; 7075-T6 SHEET, K_f = 2.62

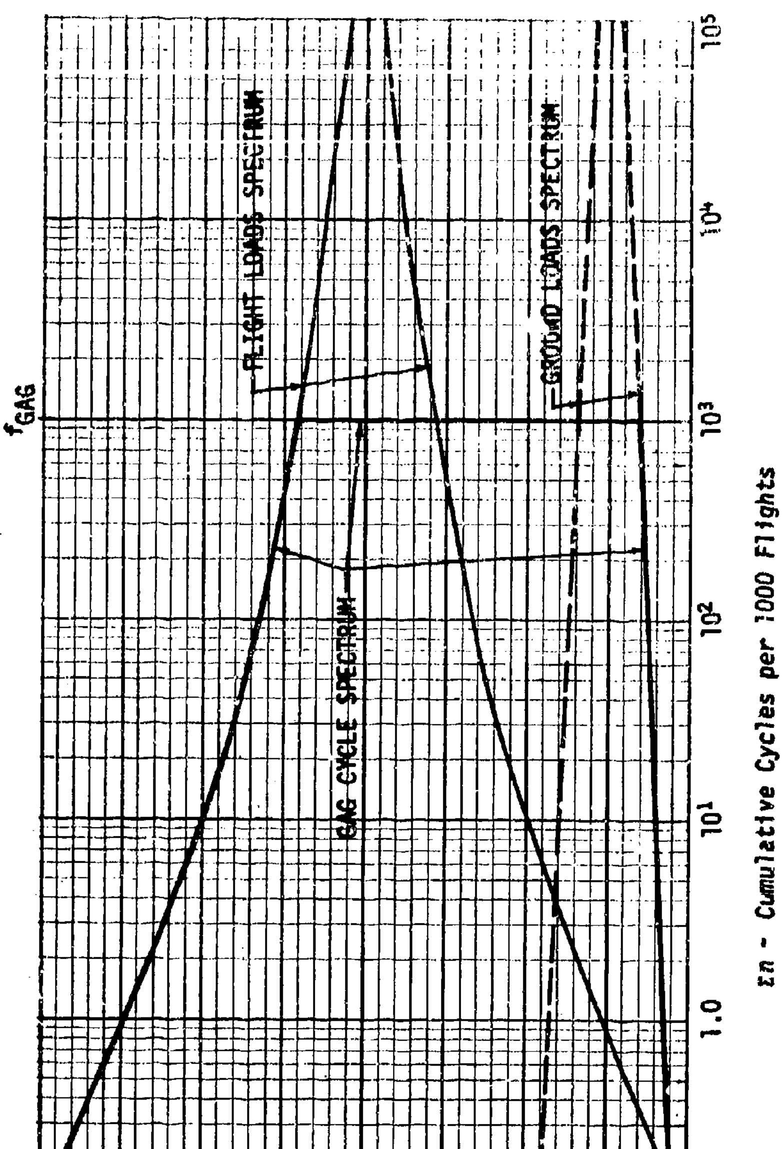


FIGURE 18. A TYPICAL AIRCRAFT STRUCTURAL ELEMENT COMPOSITE FATIGUE LOADS SPECTRUM

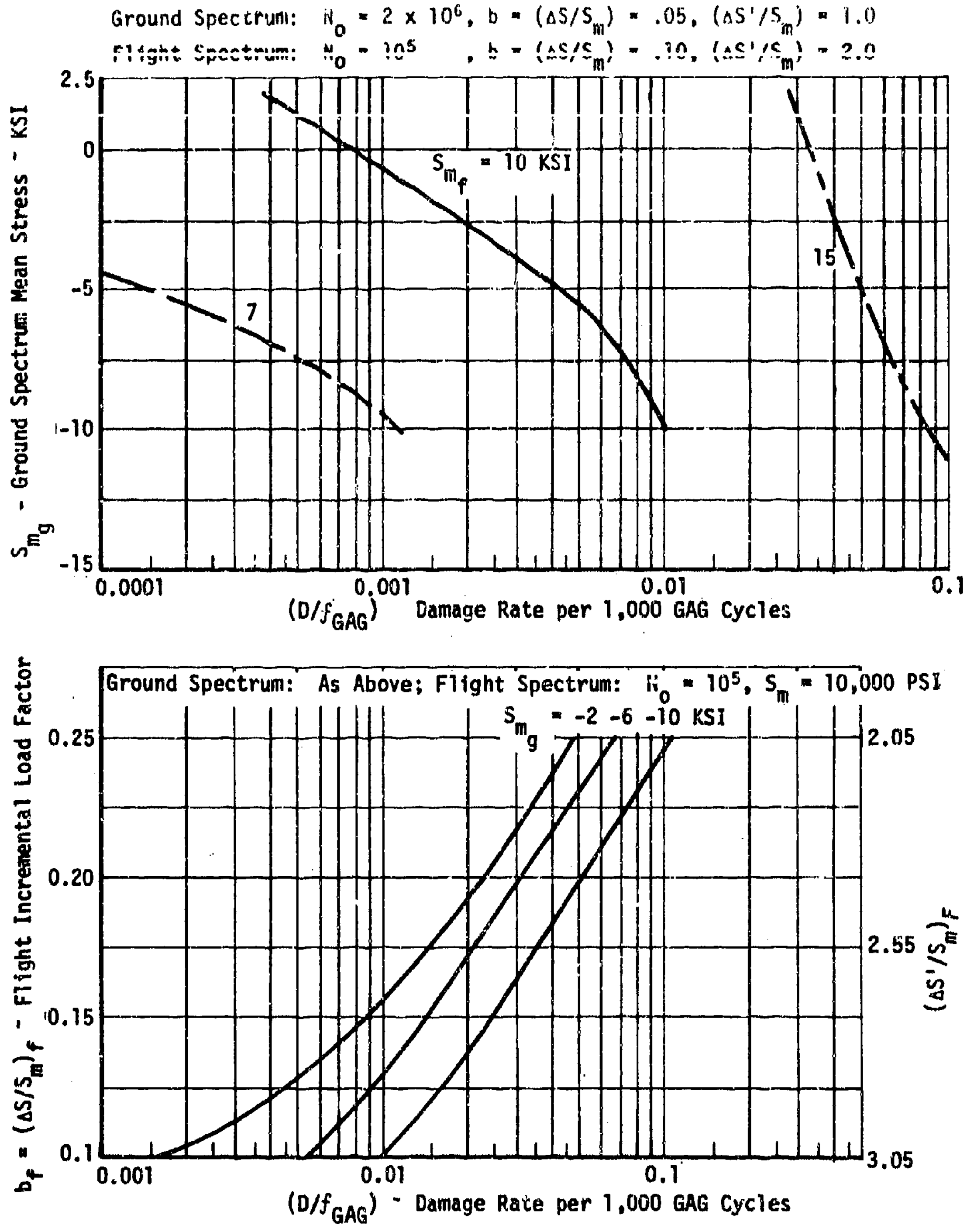
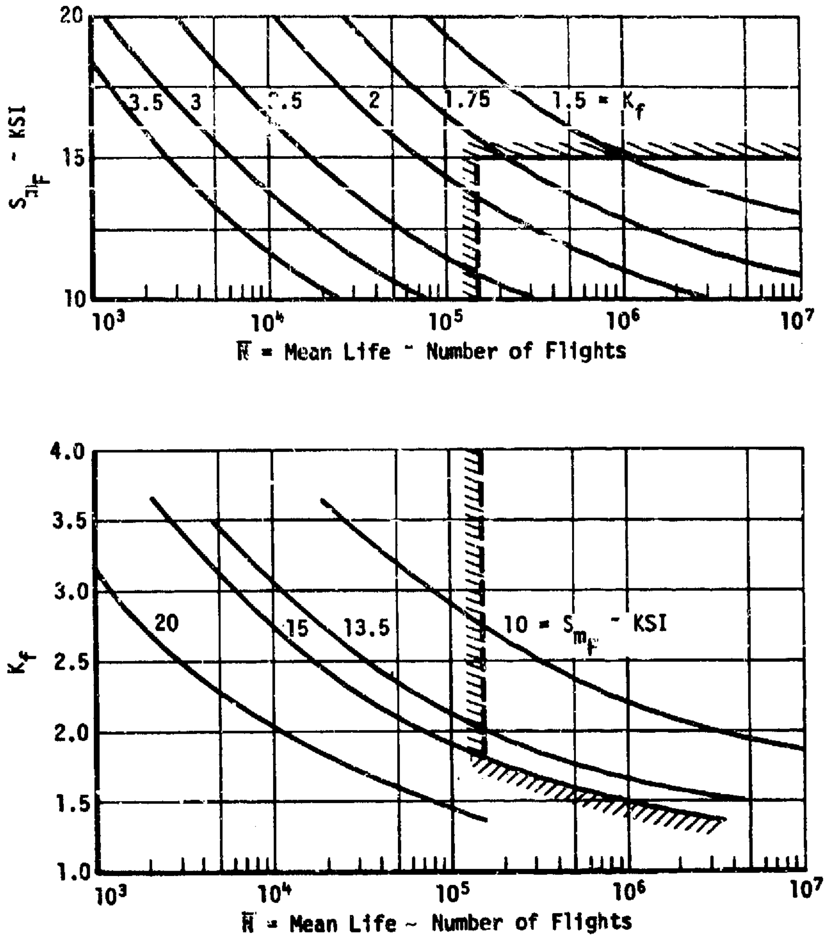


FIGURE 19. GAG CYCLE SPECTRUM DAMAGE RATES; 7075-T6 SHEET, K_f = 2.62



$S_{m_G} = -(S_{a_F}/2)$, Net Area Stress, 7075-T6 Aluminum

Design Envelope: $N \geq 150{,}000$ Flights
$S_{m_F} \leq 15{,}000$ psi, (Operational Stress)

FIGURE 20. FATIGUE STRENGTH DESIGN CHARTS FOR AIRCRAFT STRUCTURES WITH THE APPLIED LOADS SPECTRUM OF FIGURE 18.

SECTION IV

CONCLUSIONS AND RECOMMENDATIONS

Fatigue strength design criteria and analysis of aircraft structures involves many disciplines: aircraft utilization and loads environments, structural response, detail stress analysis, fatigue damage accumulation, statistical aspects of fatigue cyclic loads and life, and testing. This study investigated the problems of fatigue life scatter, specification of fatigue life design requirements, and methods for the development of general fatigue strength design charts.

Operational fatigue life scatter is a function of the basic fatigue life scatter, as exemplified by laboratory fatigue test results, and of the variation of the operational applied loads spectra among individual aircraft in a fleet of aircraft. Consequently, the probability of fatigue failure of a given element in the fleet of aircraft, at life N_j, is a joint probability problem,

$$p(N_j) = p(N_j|L_i) \times p(L_i) \tag{42}$$

where, $p(N_j|L_i)$ is the probability of failure at life N_j, given loads spectrum L_i, and $p(L_i)$ is the probability of the occurrence of the loads spectrum L_i. The probability, $p(N_j|L_i)$, is represented by the basic fatigue life scatter. A statistical evaluation was accomplished in this study of over 6,000 aluminum alloy specimen fatigue test results to define the basic fatigue life scatter magnitude and distribution. The specimens ranged in complexity from simple material unnotched and notched specimen to structural components and full-scale structures. Both constant amplitude and spectrum loading test data were considered. Based on the evaluation of this large sample of fatigue test results, the following basic fatigue life scatter properties were observed:

1. Basic fatigue life scatter distribution greatly deviates from the log Normal distribution at lives $N \gtrless \mu \pm 2\sigma$. Equations (7) and (8) represent basic fatigue scatter frequency and probability distributions as derived from the surveyed test data.

2. Scatter is greater under constant amplitude loading than under spectrum loadings.

3. In general, unnotched specimen, and to a certain extent, notched specimen, exhibit more scatter than structural components. The relatively high scatter observed in full-scale structure test results is attributed to the fact that the great majority of the specimens tested had previous actual service loading history. Therefore, the larger amount of scatter reflects not only the basic fatigue scatter, but also includes the effect of the operational loads spectra variation.

4. In general, fatigue scatter increases with increase in life, in particular, under constant amplitude loading.

5. Under spectrum loading, based on notched specimen and structural component test data, a log standard deviation of 0.14 is recommended for statistical evaluation of the basic fatigue life scatter of aluminum alloy aircraft structures in conjunction with the derived scatter distributions of equations (7) and (8).

Operational life scatter concepts, as a function of the joint probability distribution model, were illustrated by the development of several joint probability distribution models. Using this concept of operational life scatter, the failure distribution of a large sample of actual service failures was correctly predicted. It appears that the operational life probability distribution, based on the joint probability distribution of the basic fatigue scatter and applied loads variation, is a valid concept and perhaps the most promising concept in defining operational life requirements for fatigue analysis and design of aircraft structures.

Fatigue life design requirements should include a specification of a desired reliability level during the required lifetime, N_R, where reliability $R = 1 - p$, and p is the probability of failure not to be exceeded at life N_R. The structure would be designed and verified, by analysis and/or testing, for a mean life $\bar{N}_c$, where c is a selected confidence level, and $\bar{N}_c$ is related to N_R by a statistically established scatter factor, $SF|_p = \bar{N}_c/N_R = \bar{N}_c/N_p$. Recommended procedures for the calculation of such scatter factors are described in Section II of this report. For example, $SF|_p = 2$, 3, and 4, in general, correspond to approximately 6, 1, and .5% probability of failure. Therefore, the design life, N_R, specifies a time interval during which the probability of fatigue failure is an acceptably realistic low value.

The term 'time to fatigue failure' is defined as the time to crack initiation and propagation of the crack until the design ultimate static strength of the structural element is reduced. For highly notch sensitive materials and structures without redundancy with MS = 0, the time to fatigue failure would be the time to crack initiation and would not include any crack propagation time.

Analytical methods and procedures for the development of generalized fatigue strength design charts are described, and samples of such charts are presented, in Section III of this report. The loads spectra are defined in equation form and the structural element fatigue quality is measured in terms of an average K_f value. The objective of such charts is to provide the designer and fatigue analyst with rapid means of fatigue strength-life estimation.

As a consequence of the above studies and from general considerations of aircraft fatigue strength design criteria and analysis problems, future research and studies should include:

1. Further collection and statistical evaluation of aluminum alloys and other commonly used aircraft materials fatigue test data to establish their typical basic fatigue scatter magnitude and distributions.

2. Collection of operational loads spectra on individual aircraft basis and development of operational loads spectra distribution for various types of aircraft.

3. A completely acceptable universal analytic fatigue damage accumulation criteria is not available at the present time and it is doubtful whether such criteria will be available in the near future. It is proposed that a study and statistical evaluation of fatigue test data which is typical of aircraft structures and loadings would result in a statistically accurate and acceptable damage rule for types of spectrum loadings generally experienced by aircraft structures.

4. A comprehensive program of collecting and interpreting fatigue service failures. Comparison of service failure lives and distributions to the theoretically predicted values and distributions. Of course, such comparison would be subject to the availability of all pertinent information and data needed for the analytical predictions. Results of such program would verify the accuracy of theoretical predictions and would be an ideal collection of bad fatigue strength design features to be avoided in the future.

5. In conjunction with the results of item (3), development of fatigue strength damage rate-design charts for typical aircraft spectra and materials.

APPENDIX

STATISTICAL EVALUATION OF FATIGUE LIFE TEST DATA

A large amount of fatigue test data were collected and statistically interpreted for the purpose of evaluating the fatigue life scatter characteristics. Only aluminum alloys data were considered. A total of 1,180 samples, representing 6,659 specimens were collected and evaluated. The following data selection rules were followed:

1. Only samples of three specimens or more were considered.
2. A sample represents a number of identical specimen tested under the same loading.
3. Samples with mean lives less than hundred cycles were excluded.
4. In general, samples with runouts (test stopped before failure occurred) at long lives were excluded. Only in several instances of large samples one or two runout values were included.
5. In the case of specimens with previous service history, only the test life was considered. Samples were composed of specimens with approximately the same service life in terms of flight hours.
6. In the case of full-scale structures initial failure lives were used. Samples were composed only of failures of the same structural element.

The test data used in the evaluation are described in Tables 7 to 14. A large portion of the statistical data reduction was accomplished with the help of a computer program. The case numbers in these tables refer to the computer program case identification numbers. The symbols k, n and Σn represent the number of samples, sample size, and total number of specimens in one case.

1. Data Reduction and Basic Results

Initially, all data were divided into groups according to:

1. Type of Specimen:
   a. Unnotched — Material Data
   b. Notched — Simply Notched Specimen
   c. Structural Component — Structural Elements ranging from a simple lug to a complex joint.
   d. Full-Scale Structure — Large aircraft components.

2. Type of Loading:
   a. Constant Amplitude
   b. Spectrum (three load levels or more)
   c. Tension-Tension
   d. Tension-Compression

3. Mean Life Range — Cycles:
   a. 10^2-10^3
   b. 10^3-10^4
   c. 10^4-10^5
   d. 10^5-10^6
   e. 10^6-10^7
   f. $>10^7$

The following parameters were calculated for each sample.

1. $\bar{N}_i$ = Antilog $(\overline{\log N_i})$ (43)

where.

$\overline{\log N_i}$ = Mean of log lives

$= \sum_j (\log_{10} N_j)/n_i$

N_j = Cycles to failure of an individual specimen

n_i = Sample size - number of specimens

2. S_i = Biased Log Standard Deviation

$= [(\sum_j (\log N_j - \overline{\log N_i})^2)/n_i]^{1/2}$ (44)

3. Log Deviation of Individual Specimen Life:

$(\log N_j - \overline{\log N_i}) \sqrt{n_i/(n_i - 1)}$ (45)

Next, the following parameters were calculated for each group of data according to the type of specimen and loading and life interval:

1. $\bar{S}_i$ = Average of Sample Biased Log Standard Deviations

$= (\Sigma S_i)/k$ (46)

where,

k = number of samples in the group

2. σ_n = Biased Log Standard Deviation of the Pooled Data.

$= ((\Sigma S_i^2 n_i)/\Sigma n_i)^{1/2} = [(\sum_i \sum_j (\log N_j - \overline{\log N_i})^2)/\Sigma n_i]^{1/2}$ (47)

3. σ_{n-k} = Unbiased Log Standard Deviation of the Pooled Data.

$= [(\Sigma S_i^2 n_i)/((\Sigma n_i) - k)]^{1/2}$ (48)

$= [(\sum_i \sum_j (\log N_j - \overline{\log N_i})^2)/((\Sigma n_i) - k)]^{1/2}$ (49)

4. $\bar{N}_{ave}$ = Average of Sample Mean Lives — Cycles

$= (\Sigma \bar{N}_i)/k$ (50)

The above data for all the groups are summarized in Tables 15 to 18.

Furthermore, these parameters were calculated for pooled data of tension-tension and tension-compression loading groups, as shown in Tables 19 and 20. Additional parameters calculated for these sets of groups were:

1. C_v = Coefficient of Variation, see Table 21.

$$= \sigma_{n-k}/\overline{\log N}_{ave} \tag{51}$$

2. Life Scatter Distribution versus Log Deviation, see Tables 22 to 28,

where the log deviation is multiplied by $(\sqrt{n_i/(n_i - 1)})$ to reduce the bias of the sample size.

2. Interpretation of Results

There are two basic questions to be answered about the fatigue life scatter. What is the fatigue life scatter frequency or density distribution and what is the magnitude of scatter? In the following discussion an attempt is made to give some answers to these questions through the interpretation of the results obtained from the survey of the fatigue test life data.

2.1 Frequency Distribution. The most commonly used frequency distribution in the evaluation of fatigue life scatter has been the "Normal" or Gaussian distribution, with the transformation of N, cycles to failure, to $\log_{10}N$.

$$f(x) = (1/\sigma\sqrt{2\pi})e^{-(\frac{x-\mu}{\sigma})^2/2} \tag{52}$$

where, $x = \log N$

$\mu = \overline{\log N} = (\Sigma \log N)/n$

$\sigma = [\Sigma(\log N - \overline{\log N})^2/(n - 1)]^{1/2}$

Fatigue test life data usually yield approximately Normal distributions. However, in most cases the samples are small and do not indicate the frequency distribution in the extreme scatter regions corresponding to low probabilities of failure in the order of 1% or less. In the design and analysis of aircraft structures for safe life, the main interest lies in the region of relatively low probabilities of failure. Consequently, log Normal approximation of small samples of fatigue test data does not prove the validity of the Normal distribution at low probabilities of failure.

In order to check the validity of the Normal distribution in the extreme distribution ranges the log deviations of many samples were pooled into groups according to the type of specimen and loading and sample mean life. The log deviation frequency distributions of these groups are summarized in Tables 22 to 28. With further pooling of life interval group data, (groups exhibiting similar standard deviations, (σ_{n-k}), frequency distributions were plotted on Normal distribution probability paper as shown by Figure 21 to 27. The cumulative probability of failure, %, was calculated as,

$$(100) \times (\Sigma n/(\Sigma n_i + 1)) \qquad (53)$$

where, Σn = Cumulative number of specimen corresponding to a given deviation value, beginning with the smallest deviation (highest negative value).

Σn_i = Total number of specimen in the group.

In addition to the test data distributions, the Normal distribution lines, based on the calculated pooled data σ_{n-k} values, are shown for comparison. The following general observations and comparisons can be made with respect to the test data and Normal distributions:

1. The pooled test data exhibits a non-Normal distribution.

2. For all practical purposes test data distributions are symmetrical about the mean.

3. With respect to lives shorter than the mean, at extreme values the test data indicates higher probabilities of failure than the Normal distribution and lower probabilities than the Normal as lives approach the mean. The reverse is true at lives longer than the mean.

4. The transition point where the test data and Normal distributions coincide ranges approximately from 1 to 10% probability of failure at lives shorter than the mean and 1 to 10% probability of survival at lives longer than the mean. The transition point approaches the mean as the standard deviation increases.

Using these observations as guidelines to derive a fatigue life scatter distribution, all test data were pooled into four large groups according to the calculated standard deviations, σ_{n-k}, of Tables 19 and 20. The data was divided into four groups of standard deviations: less than 0.150, 0.150 to 0.200, 0.200 to 0.300, and greater than 0.300, as shown in Table 29, regardless of the type of specimens, loading or life interval. The log deviation distributions of these four groups, normalized by dividing the deviations by the calculated standard deviation, σ_{n-k}, were plotted on Normal probability paper as shown by Figures 28 to 31. Based on these four test data distributions, a three-term exponential expression was derived for the calculation of the fatigue life cumulative probability of failure distribution,

$$F(-x) = A_1 e^{-d_1 |x|} + A_2 e^{-d_2 |x|} + A_3 e^{-d_3 |x|}, \; x \le 0 \qquad (54)$$

and, $F(x) = 1 - F(-x), \; x > 0$

where, $x = (\log N - \overline{\log N})/\sigma$

$\sigma = [\Sigma(\log N - \overline{\log N})^2/(n-1)]^{1/2}$

$$A_1 = 1.687\sqrt{\sigma} \qquad d_1 = 1.3 + 0.86\sqrt{\sigma}$$

$$A_2 = 0.015 \qquad d_2 = 0.28 + 0.44\sqrt{\sigma}$$

$$A_3 = 0.485 - 1.687\sqrt{\sigma} \qquad d_3 = 1.09 + 2.16\sqrt{\sigma}$$

Use of this cumulative probability of failure expression for standard deviations, σ, greater than 0.75 is not recommended. In reality this is not a limitation since fatigue life scatter seldom exceeds a standard deviation of 0.75. The cumulative probability of failure distributions, for selected values, as calculated by Equation (54) are shown plotted in Figure 32 and in Figures 28 to 31 for comparison with the original test data. From Figure 32 it is seen that the fatigue test data probability distributions of Equation (54), regardless of the σ value, and the standard Normal distribution coincide at a probability of failure of approximately 4% corresponding to 1.75 standard deviations from the mean. Furthermore, the test data Equation (54) indicates higher probabilities of failure than Normal at standard deviations greater than 1.75 from the mean, whereas at standard deviations less than 1.75 from the mean the test data approaches the Normal distribution at the standard deviation $\sigma = 0.75$.

Since the cumulative probability is the area under the frequency (density) distribution function, differentiating Equation (54) with respect to x we obtain the frequency distribution function,

$$f(x) = \frac{d}{dx} F(-x) = -\left(A_1 d_1 e^{-d_1|x|} + A_2 d_2 e^{-d_2|x|} + A_3 d_3 e^{-d_3|x|}\right)$$

$$= -\left(C_1 e^{-d_1|x|} + D_2 e^{-d_2|x|} + C_3 d^{-d_3|x|}\right) \qquad (55)$$

where

$$C_1 = A_1 d_1,\ C_2 = A_2 d_2,\ C_3 = A_3 d_3 \text{ and } f(-x) = f(x).$$

The negative sign on the right side of equation (55) can be disregarded for all practical purposes of calculating $f(x)$. The test data frequency distribution functions as calculated by equation (55) for selected values of σ and the standard Normal distribution are shown plotted in Figure 33.

2.2 Standard Deviations. The standard deviation is the measure of fatigue life scatter with respect to the mean life. The magnitude of the standard deviation reflects the amount of dispersion of fatigue lives about the mean. This is true of the Normal frequency distribution as well as the frequency distribution expression derived from test data, see equations (52) and (55).

The calculated standard deviation values, based on the fatigue test data survey, are summarized in Tables 15 to 18, according to the type of cyclic loading, specimen, and mean life interval. Pooling of the same type of loading, specimen, and mean life small sample data into larger groups was justified on the assumption that all samples come from the same population. Following general observations can be made about the magnitude of scatter in terms of the calculated unbiased, σ_{n-k}, standard deviations:

1. No consistent trend is observable between tension-tension and tension-compression loading σ_{n-k} values, see Tables 15 to 18. Consequently, the tension-tension and tension-compression data were pooled together and the results are presented in Tables 19 and 20.

2. Scatter is proportional to life. Scatter increases with increase in life from approximately $N = 10^4$, see Figures 34 and 35.. There is also some evidence of increase in scatter as lives become relatively short. Thus, it appears that the greatest amount of scatter can be expected at the short and long lives. This can be attributed to the variability of the static ultimate strength at short lives and the statistical aspects of the fatigue strength endurance limit at long lives. The variation of the standard deviation as a function of the mean life in terms of the coefficient of variation, $C_V = \sigma_{n-k}/\log N_{ave}$, is illustrated by Figures 36 and 37. The variation of C_V with life is similar to the variation of standard deviation.

3. In general, scatter is greatest for unnotched specimen, and least for structural components. However under constant amplitude loading, notched specimen scatter exceeds that of the unnotched specimen, except at short and long lives, whereas under spectrum loading, notched specimen and structural component scatter is approximately the same.

4. The relatively high scatter of full-scale structure test lives is somewhat surprising at first. It is consistently higher than structural component scatter and sometimes exceeds the scatter of notched specimen. One would expect the scatter of structural components and full-scale structure lives to be about the same considering that the full-scale structure test life samples were defined by initial failures of the same structural element and not the final failure of the complete structure. One possible explanation of this is the fact that most of the full-scale structures tested had a previous service loading history. Although samples were composed of specimen with approximately the same service life, as measured by flight hours, the amount of damage accumulated by each specimen in service life prior to testing varied. Consequently, flight hours are not the absolute measure of the specimen life, or in effect, of the damage accumulated by the structure, the damage being the true measure of the consumed life. Thus, the relatively high scatter in test lives of full-scale structures with previous service history reflects not only the basic fatigue scatter, but also, partly, the scatter due to the variation of service loads spectra. Another factor to consider in the interpretation of full-scale test results is the probable difficulty in detecting the crack initiation consistently for each specimen. This fact could also contribute to the higher scatter exhibited by the full-scale structure test results as compared to the structural component scatter.

5. Scatter appears to be greater under constant amplitude loading than under spectrum loading when the comparison is made between the same type of specimen at the same life, see Figures 34 and 35. It should be noted that if the lives under spectrum loading were divided by approximately a factor

of ten, a much closer agreement between constant amplitude and spectrum loading standard deviations is observed. One plausible explanation of this phenomena could be the fact that often, under spectrum loading, the spectrum contains many cycles of low loads which contribute a negligible amount to the total damage. Exclusion of these low load cycles from the measure of life under spectrum loading would reduce the life, in terms of cycles, to a common basis for comparison of spectrum and constant amplitude loading lives.

3. Concluding Remarks and Recommendations

Fatigue life of materials and structures is a statistical value and for this reason the fatigue life scatter statistical model parameters must be defined. These parameters are the mean life, the frequency distribution and the standard deviation. The mean life is directly a function of the type of loading and specimen and can not be generalized. However, a standard fatigue life scatter frequency distribution, and in turn, a probability distribution, can be assumed to exist, associated with the magnitude of scatter as measured by the standard deviation. The survey made in this study of 1,180 test samples, representing 6,659 aluminum alloy specimens, ranging from unnotched specimen to full-scale structures, indicates the following results:

1. On the assumption that a common fatigue life scatter frequency distribution exists, general frequency and probability functions, equation (54) and (55), were derived as a function of the standard deviation. These expressions differ from the Normal-Gaussian distribution as shown in Figures 32 and 33.

2. The measure of the scatter about the mean, the standard deviation, was found to vary as a function of the type of loading, specimen, and mean life as illustrated by Figures 34 and 35. The magnitude and variation of the standard deviations must be considered to represent the typical fatigue life scatter under similar loading, specimen, and life conditions.

Based on the evidence of the fatigue test data survey results, the following tentative recommendations are made for the statistical interpretation of the basic fatigue life scatter of aluminum alloy materials and structures:

1. The frequency and probability distributions, equations (54) and (55) should be used in lieu of the log-Normal distribution.

2. Recommendation of basic standard deviations as a function of type of loading, specimen, and life, remains a dilemma, as exemplified in the discussions of the test data results in Section 2.2 of this appendix. More test data, and in some areas a more detailed treatment of the data are needed to clarify the discrepancies brought out in Section 2.2. Keeping in mind the need of further detail study of additional test data, following standard deviation values are recommended for use in the statistical evaluation of fatigue life scatter:

a. For the evaluation of constant amplitude S-N test data, use standard deviations presented by Figure 38. Two sets of standard deviations are presented: one for simple unnotched and notched materials specimen, the other for structural components. The simple specimen standard deviations are based on the unnotched and notched specimen constant amplitude loading pooled data. The standard deviation values for structural components, applicable to any structural element with multiple stress concentrations, are based on structural component test data with the exclusion of the $\bar{N} < 10^7$ data which appears to be unrealistic in view of all the other data, see Figure 34.

b. For the evaluation of spectrum loading test data, the standard deviations of Figure 35, in the life range $10^4 < \bar{N} < 10^7$, are recommended for simple unnotched and notched, and structural component specimen. The full-scale structure standard deviations, it must be remembered, represent not only the basic fatigue scatter, but also the scatter due to loads spectra variation as pointed out in Section 2.2 of the Appendix.

c. For the purpose of general fatigue analysis and design of aircraft structures under spectrum loading, a standard deviation of 0.14 is recommended for statistical evaluation of the basic life scatter. This value is the unbiased standard deviation of all notched specimen and structural component spectrum loading data consisting of 305 samples and 2,106 specimen.

3. Fatigue life scatter of aircraft structures in service is a function not only of the basic fatigue scatter, which can be defined as the scatter exhibited by laboratory specimen, but also a function of the loads spectrum variation in a fleet of aircraft. As noted in Section 2.2 of the Appendix, the full-scale structure test data surveyed in this study represents not only the basic fatigue scatter, but in part, also reflects the effect of service loads spectrum variation. On the basis of all full-scale structure spectrum loading test data, represented by 35 samples and 202 specimen, a standard deviation value of not less than 0.20 is recommended for use in the statistical evaluation of service life scatter of aircraft structures when the mean life is based on average operational loads spectrum. (For comparison, the standard deviation based on all full-scale structure constant amplitude loading test data, represented by 91 samples and 378 specimen, is 0.26.)

TABLE 7

FATIGUE TEST DATA DESCRIPTION
Constant Amplitude Loading — Unnotched Specimen

Case No.	Material	Loading	k	n	Σn	Ref.
1	2024-T81 Sheet	Axial	19	3,4	60	11
2	2024-T3 Sheet	Axial	18	3-6	67	11
5	7075-T6 Sheet	Axial	48	3-8	206	11
20	24S-T3 Sheet	Axial	7	3,4	25	12
21	75S-T6 Sheet	Axial	8	3,4,6	31	12
22	24S-T3 Sheet	Axial	3	4,5	14	12
23	75S-T6 Sheet	Axial	3	3	9	12
24	24S-T3 Sheet	Axial	3	3,4	10	12
25	75S-T6 Sheet	Axial	6	3,4	20	12
30	7075-T6 Extr. Rod	Rotating Beam	9	3,9-11	82	13
36	75S-T6 Hand Forg. Plate	Axial	3	3	9	15
54	7079-T6 Hand Forg. Rod	Axial	1	3	3	14
		Total:	128		536	

TABLE 8

FATIGUE TEST DATA DESCRIPTION
Constant Amplitude Loading — Notched Specimens

Case No.	Material	$K_T(K_F)$	Notch	Loading	k	n	Σn	Ref.
8	2024-T3, 2024-T81, 7075-T6 Sheet		Hole	Axial	6	3-5	23	11
13	7075-T6 Sheet	4.0	Edge	Axial	13	3-5,9	50	16
14	2024-T3 Sheet	4.0	Edge	Axial	14	3,5, 6,10	79	16
15	7075-T6 Sheet	4.0	Edge	Axial	14	4-8	74	17
16	7075-T6 Sheet	3.0	Hole	Axial	20	5	100	18
17	7075-T6 Sheet	4.0	Elipse	Axial	25	5	125	18
18	7075-T6 Sheet	7.0	Elipse	Axial	25	5	125	18
19	7075-T6 Sheet	10.0	Elipse	Axial	20	5	100	18
26	24S-T3 Sheet	2.0	Hole	Axial	1	3	3	19
27	24S-T3 Sheet	4.0	Fillet	Axial	1	3	3	19
28	75S-T3 Sheet	2.0	Edge	Axial	1	3	3	19
29	75S-T6 Sheet	4.0	Edge	Axial	1	3	3	19
31	7075-T6 Extr.Rod	1.38	Groove	Rotating Beam	6	9,10	58	13
32	7075-T6 Extr.Rod	3.0	Groove	Rotating Beam	10	9,10	98	13
33	7075-T6 Extr.Rod	5.0	Groove	Rotating Beam	8	10	80	13
34	24S-T3 Sheet	4.0	Edge	Axial	2	3,4	7	20
37-50	75S-T6 Hand Forg. Plate	(1.2-1.5)	Fillet (Lug)	Axial	54	3,4	163	15
51,52	2014-T6 Hand Forg. Rod	2.4	Groove	Axial	2	3	6	14
53	7075-T6 Hand Forg. Rod	2.4	Groove	Axial	2	3	6	14
55	2024-T3	4.0	Edge	Axial	1	6	6	21
56	7075-T6	4.0	Edge	Axial	1	5	5	21
				Total:	227		1117	

TABLE 22

FATIGUE TEST LIF SCATTER DISTRIBUTION
Constant Amplitude Loading — Unnotched Specimen

$[(\log N_i - \overline{\log N_i}) \times \sqrt{n_i/(n_i-1)}]$	No. of Specimen in the Life (Cycles) and Deviation Range					
	10^2-10^3	10^3-10^4	10^4-10^5	10^5-10^6	10^6-10^7	$>10^7$
-1.9 to -1.8						1
-1.6 -1.5					1	
-1.5 -1.4					1	1
-1.4 -1.3					1	
-1.3 -1.2						2
-1.1 -1.0				1		2
-1.0 -0.9					1	
-0.9 -0.8				2	1	
-0.8 -0.7				2	2	1
-0.7 -0.6		1	1	2		2
-0.6 -0.5			1	2		1
-0.5 -0.4	2		1	8	3	1
-0.4 -0.3	2	2	5	11	1	2
-0.3 -0.2	2	5	10	10	2	2
-0.2 -0.1	4	6	20	18		1
-0.1 to -0.0	6	18	44	29		1
0.0 to 0.1	5	31	78	19	1	6
0.1 0.2	2	8	30	23	3	4
0.2 0.3	5	1	3	8	1	1
0.3 0.4	1	4	4	8	2	3
0.4 0.5				4		2
0.5 0.6			1	2	1	1
0.6 0.7				4	2	3
0.7 0.8				2	3	
0.8 0.9	1				2	
0.9 1.0					1	
1.0 1.1				1		1
1.1 1.2				1		1
1.2 1.3				2		3
1.5 to 1.6					1	
Σn_i	30	76	198	159	30	43

TABLE 9

FATIGUE TEST DATA DESCRIPTION
Constant Amplitude Loading — Structural Components

Case No.	Specimen	Material	k	n	Σn	Ref.
3,4	Lug (Loaded Hole)	2024-T3 Sheet	66	3-6,8	263	11
6,7,9-12	Lug (Loaded Hole)	7075-T6 Sheet	65	3-6,8, 11	283	11
72	Riveted Lap Joint	7075 Clad Sheet	4	10	40	22
73	Riveted Lap Joint	2024 Clad Sheet	5	10	50	22
74	Riveted Lap Joint	2024 Clad Sheet	15	3,7	97	23
92,93	Riveted Beam	7075-T6	3	3,4	10	24
100	Fuselage Skin Joint	14S-T, 24S-T, 75S-T	25	3-6	95	25
105	Frame-Stringer A Attach.	24S-T3, 75S-T6	3	5,6,8	19	26
110	Scarf Splice	7075-T6	6	4	24	27
115	Spar Cap Splice	7075-T6	3	3,4	11	28
116	Skin-Stringer Splice	7075-T6	1	4	4	28
120	Skin-Stringer Splice	7075-T6	1	3	3	29
121	Skin Splice	7075-T6	3	3,6	12	30
125	Skin Splice	7075-T6	8	3	24	31
130	Skin-Stringer Basic Structure	7075-T6	1	4	4	32
135,136	Spar Cap Simulation Element	7075-T6	8	3-5	29	33
140	Lug	DTD 363A, 364B 683	5	3,4,7	20	34
142-144	Lap Joint	24S-T Clad Sheet	14	3,5,6	47	35
145-147	Lap Joint	75S-T Clad Sheet	17	3-6	65	35
150	Lap Joint	24S-T Clad Sheet	4	10,20	60	36
152	Landg. Gear Component	7075-T6	4	3-5,11	23	37
153	Frame-Longeron Attachm.	75S-T6	15	5	75	37
154	Antenna Attachement	75S-T6	2	3,4	7	37
155	Longeron Splice	7075-T6	2	3,5	8	37
156	Eyebolt	7075-T6	1	11	11	37
157	Latch Fitting	7075-T6	1	6	6	37
		Total:	282		1,290	

TABLE 10

FATIGUE TEST DATA DESCRIPTION
Constant Amplitude Loading — Full Scale Structures

Case No.	Specimen	Material	k	n	Σn	Ref.
75	T-29 Outer Wing	7075	18	3-6,8	83	38
77	C-46 Wing	2024	9	3-6	37	39
80,81	P-51 (Mustang) Wing	2024	57	3-8	229	40
85	Meteor Tailplane	DTD 390	2	3	6	41
90	Fighter Horiz. Tail	7075	4	4,6	20	42
91	Fighter Wing	7075	1	3	3	42
		Total:	91		378	

TABLE 11

FATIGUE TEST DATA DESCRIPTION
Spectrum Loading — Unnotched Specimens

Case No.	Spectrum	Material	k	n	Σn	Ref.
650	Sinusoidal Modulation	7075-T6 Extr.Rod	9	4-6,11	50	13
651	Exponential Modulation	7075-T6 Extr.Rod	7	3-6	27	13
660,661	Random Excitation	2024-T4 Extr.Rod	10	6,7	64	43
662	Quasi-Stationary Excitation	2024-T4 Extr.Rod	3	3,12, 16	31	43
663,664	Random Excitation-Pre-Stress	2024-T4 Extr.Rod	21	5-7	115	43
753	4-6 Step Maneuver	7075-T6 Sheet	14	3,4	51	11
780	Sinusoidal Modulation	24S-T4 Extr. Rod	14	10,11	141	44
781	Exponential Modulation	24S-T4 Extr. Rod	13	10,11	131	44
784	Exponential Modulation	2024	11	20	220	45
785	Exponential Modulation	7075	10	20	200	56
		Total:	112		1,030	

TABLE 12

FATIGUE TEST DATA DESCRIPTION
Spectrum Loading — Notched Specimen

Case No.	Material	K_T	Notch	Spectrum	k	n	Σn	Ref.
301	2024-T3 Sheet	4.0	Edge	8 Step Gust	4	6	24	47
310	7075-T6 Sheet	4.0	Edge	8 Step Gust	2	6	12	47
315	7075-T6 Sheet	4.0	Edge	8 Step Maneuver	3	6	18	47
330	7075-T6 Sheet	4.0	Edge	8 Step Gust + GAG	22	6,7	134	21
352	2024 Sheet	4.0	Edge	8 Step Gust + GAG	2	6	12	21
371	2024 Sheet	4.0	Edge	18 Step Gust	5	3,4,6, 8,9	30	16
376	2024 Sheet	4.0	Edge	8 Step Gust	8	3,6	30	16
384	7075 Sheet	4.0	Edge	8 Step Gust	13	3-6	57	16
420	7075 Sheet	4.0	Edge	4,8 Step Maneuver	10	6	60	17
450	7075 Sheet	4.0	Edge	Maneuver	10	6-8	63	48
575	7075 Sheet	4.0	Elipse	Gust, Gust + GAG	15	4,5,7, 8, 10	86	18
580	7075 Sheet	4.0	Elipse	Manv.,Manv. + GAG	4	5,7	22	18
585	7075 Sheet	7.0	Elipse	Gust, Taxi, Composite	7	5,6	40	18
629	24S-T,7178-T6,DTD 363A Extr. Rod	7.0	Groove	Gust, Gust + GAG	8	9,19, 20,30	157	49
634	DTD 363A Extr. Rod	4.0	Groove	Gust	3	5,6	17	49
636	DTD 363A Extr. Rod	3.7	Groove	Maneuver	6	3,4	19	49
652	7075-T6 Extr. Rod	3.0	Groove	Sinusoidal Modulation	8	4,5,9	45	13
653	7075-T6 Extr. Rod	3.0	Groove	Exponential Modulation	7	3-5	28	13
654	7075-T6 Extr. Rod	3.0	Groove	Gust	3	9,14,15	38	13
680	7075-T6 Sheet	4.0	Elipse	Random Gust	9	3-6,8	41	50
752	2024-T6, 7075-T6 Sheet		Hole	4-6 Step Maneuver	5	3-5	21	11
788	7075-T6 Extr. Rod	3.2	Groove	Exponential Modulation	20	10-12, 14	207	51
789	7075-T6 Extr. Rod	3.2	Groove	Exp. Modul., Pre-Stress	38	8,10	378	51
792	2024-T3 Sheet	4.0	Edge	Random Gust	15	6	90	52
793	2024-T3 Sheet	4.0	Edge	Constant Mean Blocks	20	6	120	52
794	2024-T3 Sheet	4.0	Edge	Variable Mean Blocks	6	6	36	52
				Total:	253		[illegible],785	

TABLE 13

FATIGUE TEST DATA DESCRIPTION
Spectrum Loading — Structural Components

Case No.	Specimen	Material	Spectrum	k	n	Σn	Ref.
642	Riveted Lap Joint	7075 Cl.Sh.	Gust, GAG	19	3-5,7	121	22
643	Riveted Lap Joint	2024 Cl.Sh.	Gust, GAG	7	7	49	22
645	Bolted Joint	L.65 Bar	Gust	4	3,5	16	53
692	Riveted Beam	7075-T6	Maneuver	4	3	12	24
698	Wing Spar Cap	7075-T6	Gust, GAG	2	3	6	54
750	Lug (Loaded Hole)	7075-T6	Maneuver	9	8-12	90	11
751	Lug (Loaded Hole)	2024-T3	Maneuver	5	3,4,6	21	11
760	Integral Skin-Str. Joint	7075-T6	Gust	1	3	3	18
761	Integral Skin-Str. Joint	7075-T6	Maneuver	1	3	3	18
			Total:	52		321	

TABLE 14

FATIGUE TEST DATA DESCRIPTION
Spectrum Loading — Full-Scale Structures

Case No.	Specimen	Material	Spectrum	k	n	Σn	Ref.
605	C-46 Wing	2024	Gust	7	3,5	27	55
610	C-46 Wing	2024	Gust	5	4	20	56
615	C-46 Wing	2024	Maneuver	5	3,4	18	56
626	P-51 (Mustang) Wing	2024	Gust, GAG	3	3,4,7	14	57
628	P-51 (Mustang) Wing	2024	Gust, GAG	4	9,10,13	45	49
630	Trainer (Provost) Wing		Maneuver	1	41	41	58
638	P-51 (Mustang) Wing	2024	Maneuver	3	5,6	16	59
690	Fighter Horiz. Tail	7075	Maneuver	6	3	18	42
691	Fighter Wing	7075	Maneuver	1	3	3	42
			Total:	35		202	

TABLE 15

FATIGUE TEST LIFE SCATTER — STANDARD DEVIATIONS
Constant Amplitude Tension-Tension Loading

Cycle Range	Specimen	k	Σn	$\bar{\bar{S}}$	σ_n	σ_{n-k}	$\bar{N}_{ave}$
10^2-10^3	Notched	9	45	.081	.090	.100	604
	Structural Component	11	46	.480	.694	.796	475
	Full-Scale Structure	6	21	.249	.264	.312	493
10^3-10^4	Unnotched	10	41	.083	.111	.127	4,480
	Notched	17	83	.104	.118	.132	4,950
	Structural Component	30	138	.115	.179	.203	4,330
	Full-Scale Structure	15	65	.249	.281	.320	3,690
10^4-10^5	Unnotched	28	107	.105	.132	.154	4.62×10^4
	Notched	20	92	.129	.167	.188	3.18×10^4
	Structural Component	66	289	.089	.107	.121	4.05×10^4
	Full-Scale Component	24	107	.161	.185	.210	3.72×10^4
10^5-10^6	Unnotched	15	61	.290	.395	.454	2.29×10^5
	Notched	15	73	.328	.402	.451	2.40×10^5
	Structural Component	88	413	.141	.169	.190	2.96×10^5
	Full-Scale Structure	14	56	.142	.156	.181	3.97×10^5
10^6-10^7	Unnotched	4	17	.590	.663	.772	2.34×10^6
	Notched	7	33	.443	.588	.663	3.05×10^6
	Structural Component	24	144	.243	.275	.302	3.27×10^6
	Full-Scale Structure	4	14	.130	.160	.189	2.35×10^6
$>10^7$	Notched	4	20	.362	.527	.589	3.22×10^7

TABLE 16

FATIGUE TEST LIFE SCATTER — STANDARD DEVIATIONS
Constant Amplitude Tension-Compression Loading

Cycle Range	Specimen	k	Σn	$\bar{S}$	σ_n	σ_{n-k}	$\bar{N}_{ave}$
10^2–10^3	Unnotched	7	30	.197	.222	.253	323
	Notched	32	146	.112	.139	.156	317
	Structural Component	6	27	.224	.309	.350	563
	Full-Scale Structure	3	11	.258	.259	.303	354
10^3–10^4	Unnotched	10	35	.135	.167	.198	5,280
	Notched	47	187	.113	.159	.183	3,920
	Structural Component	28	119	.084	.103	.118	4,440
	Full-Scale Structure	9	37	.193	.222	.255	4,350
10^4–10^5	Unnotched	26	91	.097	.121	.143	4.38×10^4
	Notched	39	176	.158	.229	.260	4.08×10^4
	Structural Component	14	56	.084	.105	.122	4.19×10^4
	Full-Scale Structure	6	28	.235	.276	.311	3.69×10^4
10^5–10^6	Unnotched	21	98	.211	.260	.293	3.78×10^5
	Notched	18	105	.378	.442	.486	4.37×10^5
	Structural Component	11	44	.101	.121	.140	4.27×10^5
	Full-Scale Structure	9	36	.211	.264	.304	3.46×10^5
10^6–10^7	Unnotched	2	13	.791	.697	.758	3.11×10^6
	Notched	11	81	.511	.521	.560	3.11×10^6
	Structural Component	4	14	.137	.144	.171	1.58×10^6
	Full-Scale Structure	1	3	.055	.055	.067	4.20×10^6
$>10^7$	Unnotched	5	43	.567	.705	.750	2.52×10^8
	Notched	8	76	.573	.660	.697	1.65×10^8

TABLE 17

FATIGUE TEST LIFE SCATTER — STANDARD DEVIATIONS
Spectrum Tension-Tension Loading

Cycle Range	Specimen	k	Σn	$\bar{\bar{S}}$	σ_n	σ_{n-k}	$\bar{N}_{ave}$
10^3-10^4	Notched	6	19	.067	.092	.111	2,705
10^4-10^5	Unnotched	1	4	.061	.061	.070	5.66×10^4
	Notched	15	89	.080	.104	.114	4.68×10^4
	Structural Component	9	57	.081	.096	.105	7.17×10^4
	Full-Scale Structure	11	36	.137	.176	.212	5.2×10^4
10^5-10^6	Unnotched	12	79	.104	.127	.138	4.17×10^5
	Notched	11	62	.068	.077	.085	1.33×10^5
	Structural Component	14	92	.103	.124	.134	5.98×10^5
	Full-Scale Structure	3	14	.163	.156	.176	5.42×10^5
10^6-10^7	Unnotched	2	14	.128	.129	.139	1.85×10^6
	Structural Component	13	72	.122	.162	.179	4.88×10^6
	Full-Scale Structure	12	47	.138	.156	.181	3.80×10^6
$>10^7$	Structural Component	3	19	.135	.135	.147	1.54×10^7

TABLE 18

FATIGUE TEST LIFE SCATTER — STANDARD DEVIATIONS
Spectrum Tension-Compression Loading

Cycle Range	Specimen	k	Σn	$\bar{S}$	σ_n	σ_{n-k}	$\bar{N}_{ave}$
10^3-10^4	Unnotched	5	19	.185	.237	.276	7,800
10^4-10^5	Unnotched	21	195	.142	.155	.164	4.47×10^4
	Notched	76	539	.056	.065	.070	5.08×10^4
	Structural Component	4	24	.114	.120	.132	5.25×10^4
	Full-Scale Structure	1	6	.270	.270	.296	9.98×10^4
10^5-10^6	Unnotched	31	324	.156	.161	.170	3.77×10^5
	Notched	114	874	.101	.139	.150	3.28×10^5
	Structural Component	6	36	.088	.130	.142	4.33×10^5
	Full-Scale Structure	7	90	.165	.180	.187	4.44×10^5
10^6-10^7	Unnotched	31	321	.271	.265	.279	3.29×10^6
	Notched	26	178	.137	.181	.196	2.72×10^6
	Structural Component	2	14	.109	.110	.119	4.09×10^6
	Full-Scale Structure	1	9	.214	.214	.227	2.20×10^6
$>10^7$	Unnotched	9	74	.497	.472	.504	2.30×10^7
	Notched	5	24	.315	.347	.390	4.44×10^7
	Structural Component	1	7	.077	.077	.083	3.64×10^7

TABLE 19

FATIGUE TEST LIFE SCATTER — STANDARD DEVIATIONS
Constant Amplitude Tension-Tension and Tension-Compression Loading

Cycle Range	Specimen	k	Σn	$\bar{S}$	σ_n	σ_{n-k}	$\bar{N}_{ave}$
10^2-10^3	Unnotched	7	30	.197	.222	.253	323
	Notched	41	191	.105	.128	.145	536
	Structural Component	17	73	.390	.582	.664	506
	Full-Scale Structure	9	32	.252	.262	.309	447
10^3-10^4	Unnotched	20	76	.109	.140	.163	4,880
	Notched	64	270	.111	.147	.169	4,200
	Structural Component	58	257	.100	.149	.169	4,380
	Full-Scale Structure	24	102	.228	.261	.298	3,940
10^4-10^5	Unnotched	54	198	.101	.127	.143	4.5×10^4
	Notched	59	268	.148	.210	.238	3.78×10^4
	Structural Component	80	345	.088	.106	.121	4.07×10^4
	Full-Scale Structure	30	135	.176	.207	.235	3.71×10^4
10^5-10^6	Unnotched	36	159	.244	.318	.362	3.16×10^5
	Notched	33	178	.356	.426	.472	3.49×10^5
	Structural Component	99	457	.137	.165	.186	3.11×10^5
	Full-Scale Structure	23	92	.169	.205	.237	3.77×10^5
10^6-10^7	Unnotched	6	30	.657	.678	.758	2.6×10^6
	Notched	18	114	.484	.541	.590	3.09×10^6
	Structural Component	28	158	.228	.266	.294	3.05×10^6
	Full-Scale Structure	5	17	.115	.147	.174	2.72×10^6
$>10^7$	Unnotched	5	43	.567	.705	.750	2.52×10^8
	Notched	12	96	.502	.634	.678	1.21×10^8

TABLE 20

FATIGUE TEST LIFE SCATTER — STANDARD DEVIATIONS
Spectrum Tension-Tension and Tension-Compression Loading

Cycle Range	Specimen	k	Σn	$\bar{S}$	σ_n	σ_{n-k}	$\bar{N}_{ave}$
10^3-10^4	Unnotched	5	19	.185	.237	.276	7,800
	Notched	6	19	.067	.092	.111	2,700
10^4-10^5	Unnotched	22	199	.139	.154	.163	4.52×10^4
	Notched	91	628	.060	.072	.077	5.01×10^4
	Structural Component	13	81	.091	.104	.113	6.55×10^4
	Full-Scale Structure	12	42	.148	.193	.228	5.65×10^4
10^5-10^6	Unnotched	43	403	.141	.155	.164	3.88×10^5
	Notched	125	936	.098	.136	.146	3.11×10^5
	Structural Component	20	128	.098	.125	.137	5.49×10^5
	Full-Scale Structure	10	104	.164	.178	.187	4.73×10^5
10^6-10^7	Unnotched	33	335	.262	.261	.275	3.2×10^6
	Notched	26	178	.137	.181	.196	2.72×10^6
	Structural Component	15	86	.120	.155	.170	4.77×10^6
	Full-Scale Structure	13	56	.144	.167	.190	3.68×10^6
$>10^7$	Unnotched	9	74	.497	.472	.504	2.3×10^7
	Notched	5	24	.315	.347	.390	4.44×10^7
	Structural Component	4	26	.121	.122	.133	2.02×10^7

TABLE 21

FATIGUE TEST LIFE COEFFICIENT OF VARIATION, C_v

Cycle Range	Specimen	Constant Amplitude k	Σn	σ_{n-k}	N_{ave}	C_v	Spectrum k	Σn	σ_{n-k}	$\bar{N}_{ave}$	C_v
10^2-10^3	Unnotched	7	30	.253	323	.101	-	-	-	-	-
	Notched	41	191	.145	536	.053	-	-	-	-	-
	Structural Component	17	73	.664	506	.246	-	-	-	-	-
	Full-Scale Structure	9	32	.309	447	.117	-	-	-	-	-
10^3-10^4	Unnotched	20	76	.163	4,880	.044	5	19	.276	7,800	.071
	Notched	64	270	.169	4,200	.047	6	19	.111	2,700	.032
	Structural Component	58	257	.169	4,380	.046	-	-	-	-	-
	Full-Scale Structure	24	102	.298	3,940	.083	-	-	-	-	-
10^4-10^5	Unnotched	54	198	.141	4.5×10^4	.031	22	199	.163	4.52×10^4	.035
	Notched	59	268	.238	3.78×10^4	.052	91	628	.077	5.01×10^4	.016
	Structural Component	80	345	.121	4.07×10^4	.026	13	81	.113	6.55×10^4	.023
	Full-Scale Structure	30	135	.235	3.71×10^4	.052	12	42	.228	5.65×10^4	.048
10^5-10^6	Unnotched	36	159	.362	3.16×10^5	.066	43	403	.164	3.88×10^5	.029
	Notched	33	178	.472	3.49×10^5	.085	125	936	.146	3.11×10^5	.027
	Structural Component	99	457	.186	3.11×10^5	.034	20	128	.137	5.49×10^5	.024
	Full-Scale Structure	23	92	.237	3.77×10^5	.043	10	104	.187	4.73×10^5	.033
10^6-10^7	Unnotched	6	30	.758	2.6×10^6	.118	33	335	.275	3.2×10^6	.042
	Notched	18	114	.590	3.09×10^6	.091	26	178	.196	2.72×10^6	.030
	Structural Component	28	158	.294	3.05×10^6	.045	15	86	.170	4.77×10^6	.025
	Full-Scale Structure	5	17	.174	2.72×10^6	.027	13	56	.190	3.68×10^6	.029
$>10^7$	Unnotched	5	43	.750	2.52×10^8	.089	9	74	.504	2.3×10^7	.068
	Notched	12	96	.678	1.21×10^8	.084	5	24	.390	4.44×10^7	.051
	Structural Component	-	-	-	-	-	4	26	.133	2.02×10^7	.018

TABLE 23

FATIGUE TEST LIFE SCATTER DISTRIBUTION
Constant Amplitude Loading — Notched Specimen

$[(\log N_i - \overline{\log N_i}) \times \sqrt{n_i/(n_i-1)}]$	No. of Specimen in the Life (Cycle) and Deviation Range					
	10^2-10^3	10^3-10^4	10^4-10^5	10^5-10^6	10^6-10^7	$>10^7$
-2.0 to -1.9						1
-1.7 -1.6					1	
-1.6 -1.5						2
-1.5 -1.4						2
-1.4 -1.3					2	1
-1.3 -1.2					1	4
-1.2 -1.1					1	
-1.1 -1.0			1		1	
-1.0 -0.9				1	1	2
-0.9 -0.8					4	
-0.8 -0.7		1	1	3	3	2
-0.7 -0.6		1	1	8	5	1
-0.6 -0.5	1	1	3	6	2	3
-0.5 -0.4		2	3	8	2	3
-0.4 -0.3	4	10	8	18	4	2
-0.3 -0.2	9	9	16	18	8	2
-0.2 -0.1	22	27	36	23	8	8
-0.1 -0.0	55	75	72	20	14	5
0.0 0.1	66	87	66	20	13	8
0.1 0.2	21	36	32	9	11	7
0.2 0.3	9	13	8	9	2	9
0.3 0.4	2	4	9	10	7	12
0.4 0.5	1	1	[illegible]	3	4	6
0.5 0.6			2	4	2	5
0.6 0.7		1	4	2	3	1
0.7 0.8	1	1	2	3	5	
0.8 0.9		1		4	4	2
0.9 1.0			[illegible]	2		6
1.0 1.1					1	
1.1 1.2				2	2	2
1.2 1.3					1	
1.4 1.5				2	1	
1.5 1.6				1		
1.6 1.7				2		
2.2 to 2.3					1	
Σn_i	191	270	268	1[illegible]	[illegible]14	9[illegible]

TABLE 24

FATIGUE TEST LIFE SCATTER DISTRIBUTION
Constant Amplitude Loading — Structural Components

$[(\log N_j - \overline{\log N_j}) \times \sqrt{n_j/(n_j-1)}]$	No. of Specimen in the Life (Cycles) and Deviation Range				
	10^2-10^3	10^3-10^4	10^4-10^5	10^5-10^6	10^6-10^7
-2.1 to -2.0	2				
-1.5 -1.4		1			
-1.4 -1.3	3				
-1.3 -1.2	1				
-1.2 -1.1	1				
-1.1 -1.0		1			
-1.0 -0.9	1				
-0.8 -0.7	1			1	
-0.7 -0.6	1			2	3
-0.6 -0.5	1			1	2
-0.5 -0.4	1	1	1	3	4
-0.4 -0.3	2	1	3	15	12
-0.3 -0.2		6	11	30	10
-0.2 -0.1	8	34	46	58	24
-0.1 -0.0	10	79	108	123	33
0.0 0.1	14	93	112	123	28
0.1 0.2	4	24	49	51	13
0.2 0.3	5	12	13	28	7
0.3 0.4	1	2	1	12	6
0.4 0.5	2	1	1	4	5
0.5 0.6	2	1		2	5
0.6 0.7	2	1		2	1
0.7 0.8	4				1
0.8 0.9	2			1	3
0.9 1.0	3				1
1.1 1.2	1			1	
1.2 to 1.3	1				
Σn_j	73	257	345	457	158

TABLE 25

FATIGUE TEST LIFE SCATTER DISTRIBUTION
Constant Amplitude Loading — Full-Scale Structures

$[(\log N_j - \overline{\log N_j}) \times \sqrt{n_j/(n_j-1)}]$	No. of Specimen in the Life (Cycles) and Deviation Range				
	10^2-10^3	10^3-10^4	10^4-10^5	10^5-10^6	10^6-10^7
-1.0 to -0.9		1			
-0.8 -0.7		1	1	1	
-0.7 -0.6	1	4	1		
-0.6 -0.5	2	1	1	2	
-0.5 -0.4	3	4	5	2	
-0.4 -0.3	1	6	9	6	1
-0.3 -0.2	1	5	6	5	2
-0.2 -0.1	3	8	20	9	
-0.1 -0.0	2	11	20	17	6
0.0 0.1	9	25	23	20	4
0.1 0.2	2	10	27	15	2
0.2 0.3	1	12	11	7	2
0.3 0.4	3	9	7	3	
0.4 0.5	3	3	2	4	
0.5 0.6	1			1	
0.6 0.7		1	2		
0.7 to 0.8		1			
Σn_j	32	102	135	92	17

TABLE 26

FATIGUE TEST LIFE SCATTER DISTRIBUTION
Spectrum Loading — Unnotched Specimen

$[(\log N_i - \overline{\log N_i}) \times \sqrt{n_i/(n_i-1)}]$	No. of Specimen in the Life (Cycles) and Deviation Range				
	10^3-10^4	10^4-10^5	10^5-10^6	10^6-10^7	$>10^7$
-1.5 to -1.4					1
-1.4 -1.3				1	
-1.3 -1.2					1
-1.1 -1.0					1
-1.0 -0.9		1	1		1
-0.9 -0.8					2
-0.8 -0.7				1	1
-0.7 -0.6		2		5	1
-0.6 -0.5	2	2	1	2	2
-0.5 -0.4		1	4	13	2
-0.4 -0.3		1	6	20	2
-0.3 -0.2	1	3	31	24	9
-0.2 -0.1	3	16	45	42	5
-0.1 -0.0	4	66	113	69	6
0.0 0.1	1	74	109	49	15
0.1 0.2	6	21	49	42	3
0.2 0.3		5	29	32	6
0.3 0.4	1	3	13	17	3
0.4 0.5		3	1	7	2
0.5 0.6	1	1		5	1
0.6 0.7			1	3	1
0.7 0.8					3
0.9 1.0				1	[illegible]
1.0 1.1					2
1.1 1.2				1	1
1.7 to 1.8				1	
Σn_i	19	199	403	335	74

TABLE 27

FATIGUE TEST LIFE SCATTER DISTRIBUTION
Spectrum Loading — Notched Specimen

$[(\log N_i - \overline{\log N_i}) \times \sqrt{n_i/(n_i-1)}]$	No. of Specimen in the Life (Cycles) and Deviation Range				
	10^3-10^4	10^4-10^5	10^5-10^6	10^6-10^7	$>10^7$
-1.1 to -1.0			1		
-0.9 -0.8			1	1	
-0.8 -0.7			2		
-0.7 -0.6			1		1
-0.6 -0.5			1	1	1
-0.5 -0.4		2	5	2	1
-0.4 -0.3		1	10	3	3
-0.3 -0.2	1	2	28	14	2
-0.2 -0.1	2	31	100	18	3
-0.1 -0.0	8	279	317	50	1
0.0 0.1	6	280	317	54	4
0.1 0.2	1	28	95	15	1
0.2 0.3		3	32	9	3
0.3 0.4	1		16	3	
0.4 0.5		1	5	4	
0.5 0.6			1	2	2
0.6 0.7		1	3	2	1
0.7 0.8			1		
0.8 to 0.9					1
Σn_i	19	628	936	178	24

TABLE 28

FATIGUE TEST LIFE SCATTER DISTRIBUTION
Spectrum Loading — Structural Components and Full-Scale Structures

$[(\log N_i - \overline{\log N_i}) \times \sqrt{n_i/(n_i-1)}]$	No. of Specimen in the Life (Cycles) and Deviation Range						
	Structural Components				Full-Scale Structures		
	10^4-10^5	10^5-10^6	10^6-10^7	$>10^7$	10^4-10^5	10^5-10^6	10^6-10^7
-0.9 to -0.8		1					
-0.7 -0.6					1		
-0.6 -0.5			1		1	2	
-0.5 -0.4						3	
-0.4 -0.3			2		2	2	4
-0.3 -0.2	4	4	1	1	1	6	4
-0.2 -0.1	8	18	15	5	7	13	10
-0.1 -0.0	31	41	29	7	4	20	11
0.0 0.1	26	43	21	7	14	26	11
0.1 0.2	7	17	10	4	6	20	6
0.2 0.3	4	3	2	2	1	8	6
0.3 0.4	1		2		3	3	3
0.4 0.5		1	2		2		1
0.5 0.6						1	
0.6 to 0.7			1				
Σn_i	81	128	86	26	42	104	56

TABLE 29

GROUPING OF TEST DATA ACCORDING TO THE STANDARD DEVIATION MAGNITUDE

σ_{n-k} Range	σ_{n-k}	k	Σn	$\bar{N}$	Loading	Specimen
.077-.150	.077	91	628	10^4-10^5	Spectrum	Notched
	.111	6	19	10^3-10^4	Spectrum	Notched
	.113	13	81	10^4-10^5	Spectrum	Structr. Comp.
	.121	80	345	10^4-10^5	Const. Ampl.	Structr. Comp.
	.133	4	26	$>10^7$	Spectrum	Structr. Comp.
	.137	20	128	10^5-10^6	Spectrum	Structr. Comp.
	.143	54	198	10^4-10^5	Const. Ampl.	Unnotched
	.145	41	191	10^2-10^3	Const. Ampl.	Notched
	.146	125	936	10^5-10^6	Spectrum	Notched
Total	.127	434	2,552			
.150-.200	.163	22	199	10^4-10^5	Spectrum	Unnotched
	.163	20	76	10^3-10^5	Const. Ampl.	Unnotched
	.164	43	403	10^5-10^6	Spectrum	Unnotched
	.169	64	270	10^3-10^5	Const. Ampl.	Notched
	.169	58	257	10^3-10^4	Const. Ampl.	Structr. Comp.
	.170	15	86	10^6-10^7	Spectrum	Structr. Comp.
	.174	5	17	10^6-10^7	Const. Ampl.	Full-Scale
	.186	99	457	10^5-10^6	Const. Ampl.	Structr. Comp.
	.187	10	104	10^5-10^6	Spectrum	Full-Scale
	.190	13	56	10^6-10^7	Spectrum	Full-Scale
	.196	26	178	10^6-10^7	Spectrum	Notched
Total	.175	375	2,103			
.200-.300	.228	12	42	10^4-10^5	Spectrum	Full-Scale
	.235	30	135	10^4-10^5	Const. Ampl.	Full-Scale
	.237	23	92	10^5-10^6	Const. Ampl.	Full-Scale
	.238	59	268	10^4-10^5	Const. Ampl.	Notched
	.253	7	30	10^2-10^3	Const. Ampl.	Unnotched
	.275	33	335	10^6-10^7	Spectrum	Unnotched
	.276	5	19	10^3-10^4	Spectrum	Unnotched
	.294	28	158	10^6-10^7	Const. Ampl.	Structr. Comp.
	.298	24	102	10^3-10^4	Const. Ampl.	Full-Scale
Total	.263	221	1,181			
.300-.758	.309	9	32	10^2-10^3	Const. Ampl.	Full-Scale
	.362	36	159	10^5-10^6	Const. Ampl.	Unnotched
	.390	5	24	$>10^7$	Spectrum	Notched
	.472	33	178	10^5-10^6	Const. Ampl.	Notched
	.504	9	74	$>10^7$	Spectrum	Unnotched
	.590	18	114	10^6-10^7	Const. Ampl.	Notched
	.664	17	73	10^2-10^3	Const. Ampl.	Structr. Comp.
	.678	12	96	$>10^7$	Const. Ampl.	Notched
	.750	5	43	$>10^7$	Const. Ampl.	Unnotched
	.758	6	30	10^6-10^7	Const. Ampl.	Unnotched
Total	.548	150	823			

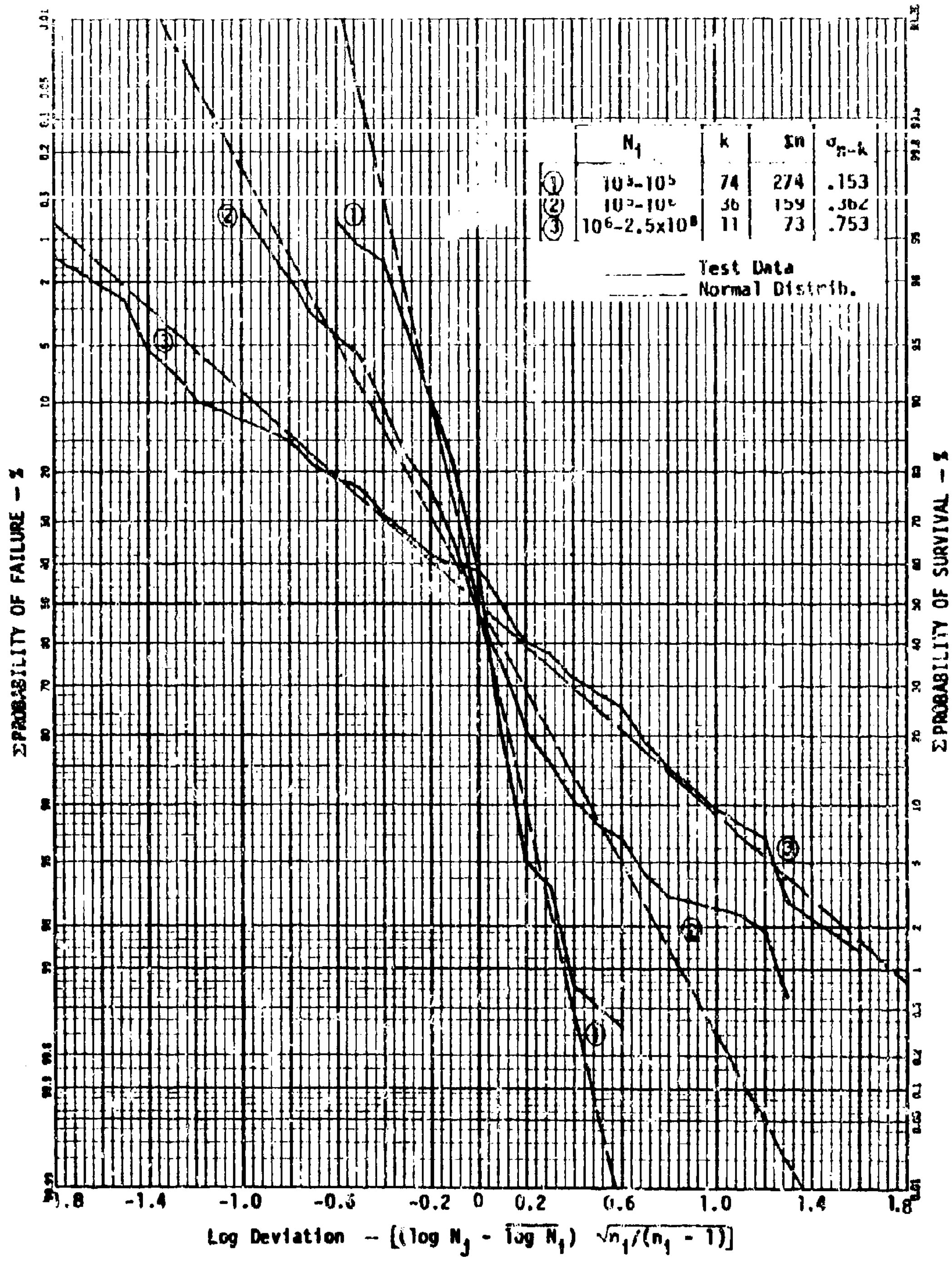


FIGURE 21. PROBABILITY DISTRIBUTIONS OF CONSTANT AMPLITUDE LOADING UNNOTCHED SPECIMEN TEST LIVES.

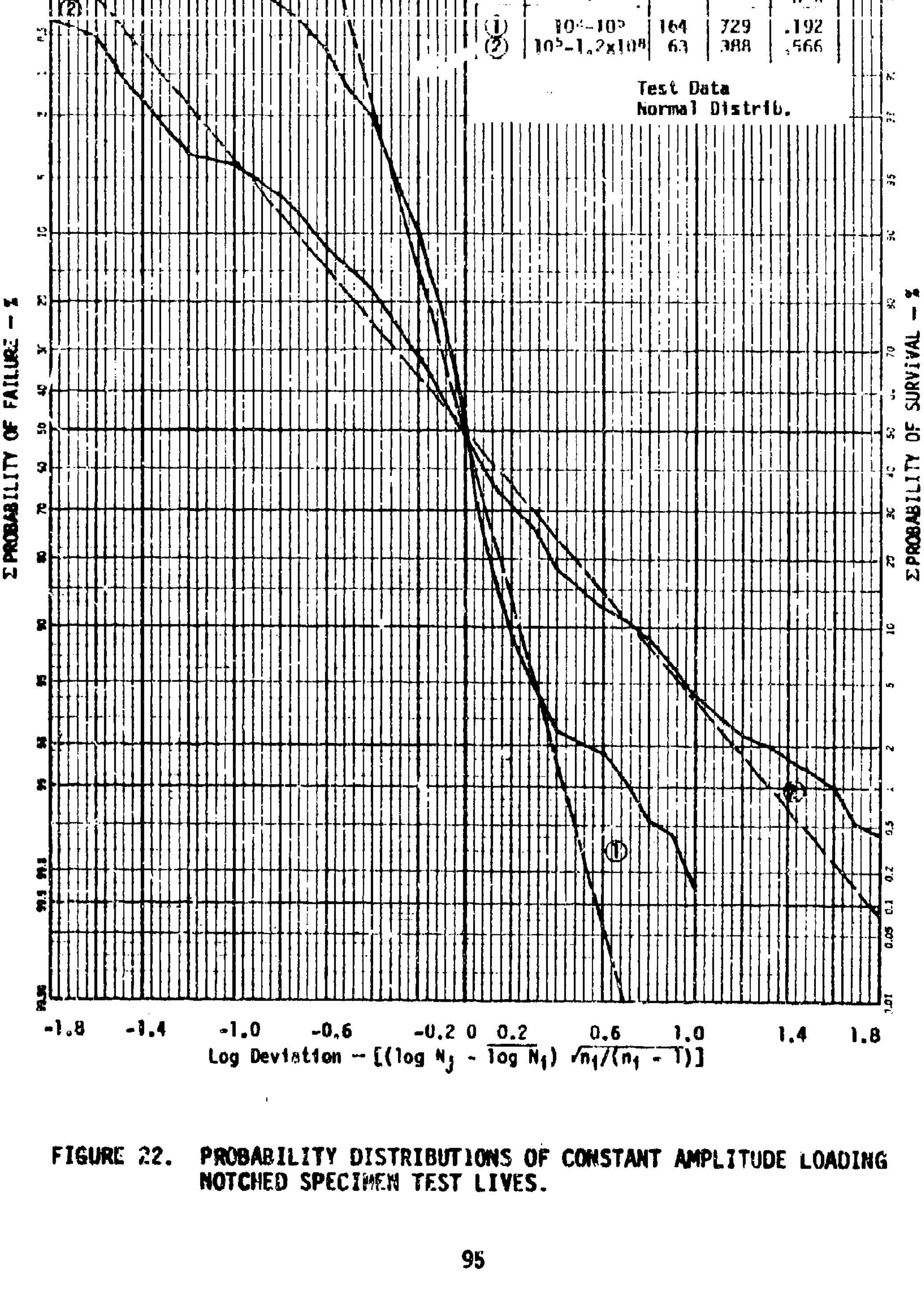


FIGURE 22. PROBABILITY DISTRIBUTIONS OF CONSTANT AMPLITUDE LOADING NOTCHED SPECIMEN TEST LIVES.

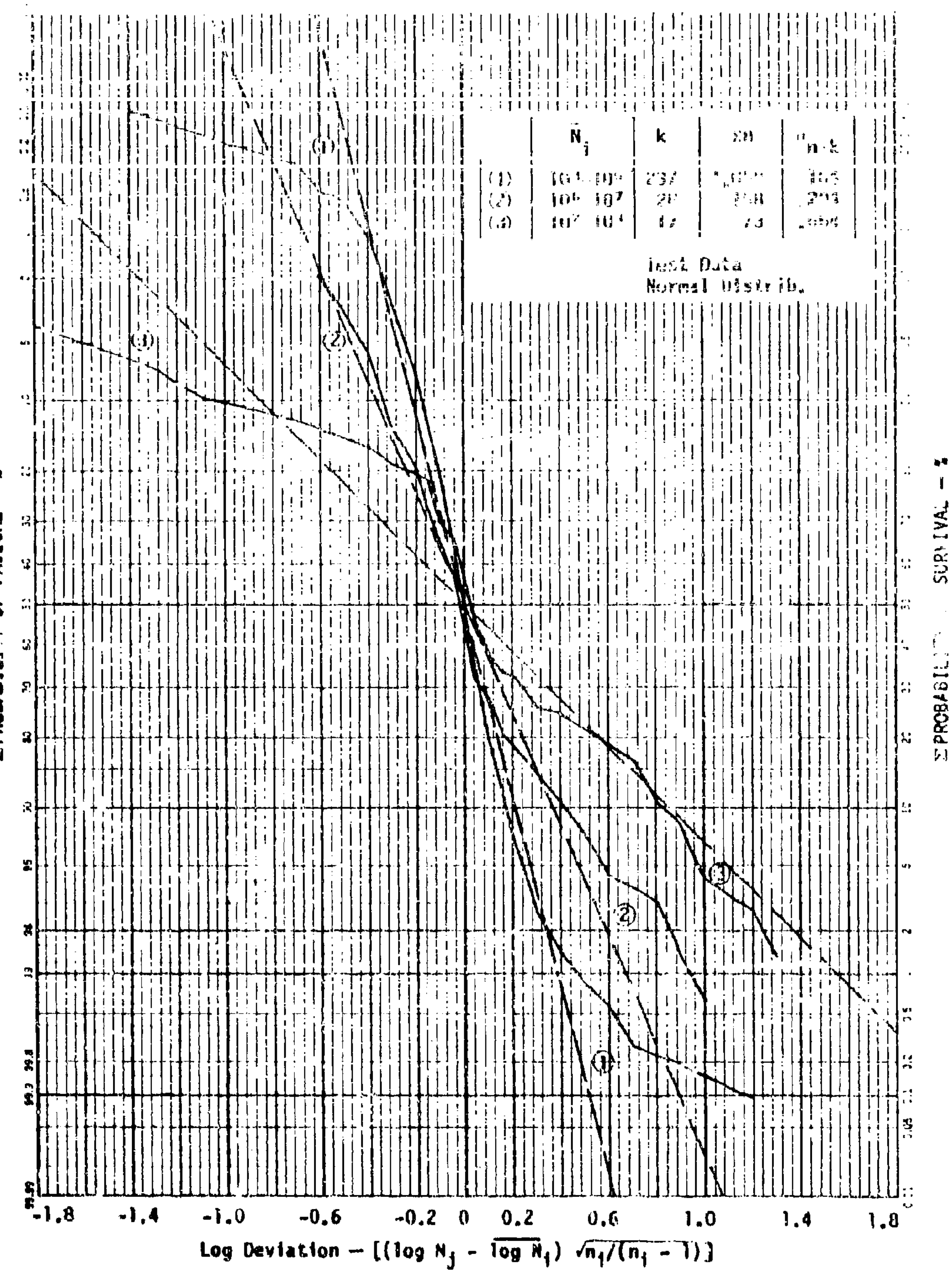


FIGURE 23. PROBABILITY DISTRIBUTIONS OF CONSTANT AMPLITUDE LOADING STRUCTURAL COMPONENT SPECIMEN TEST LIVES.

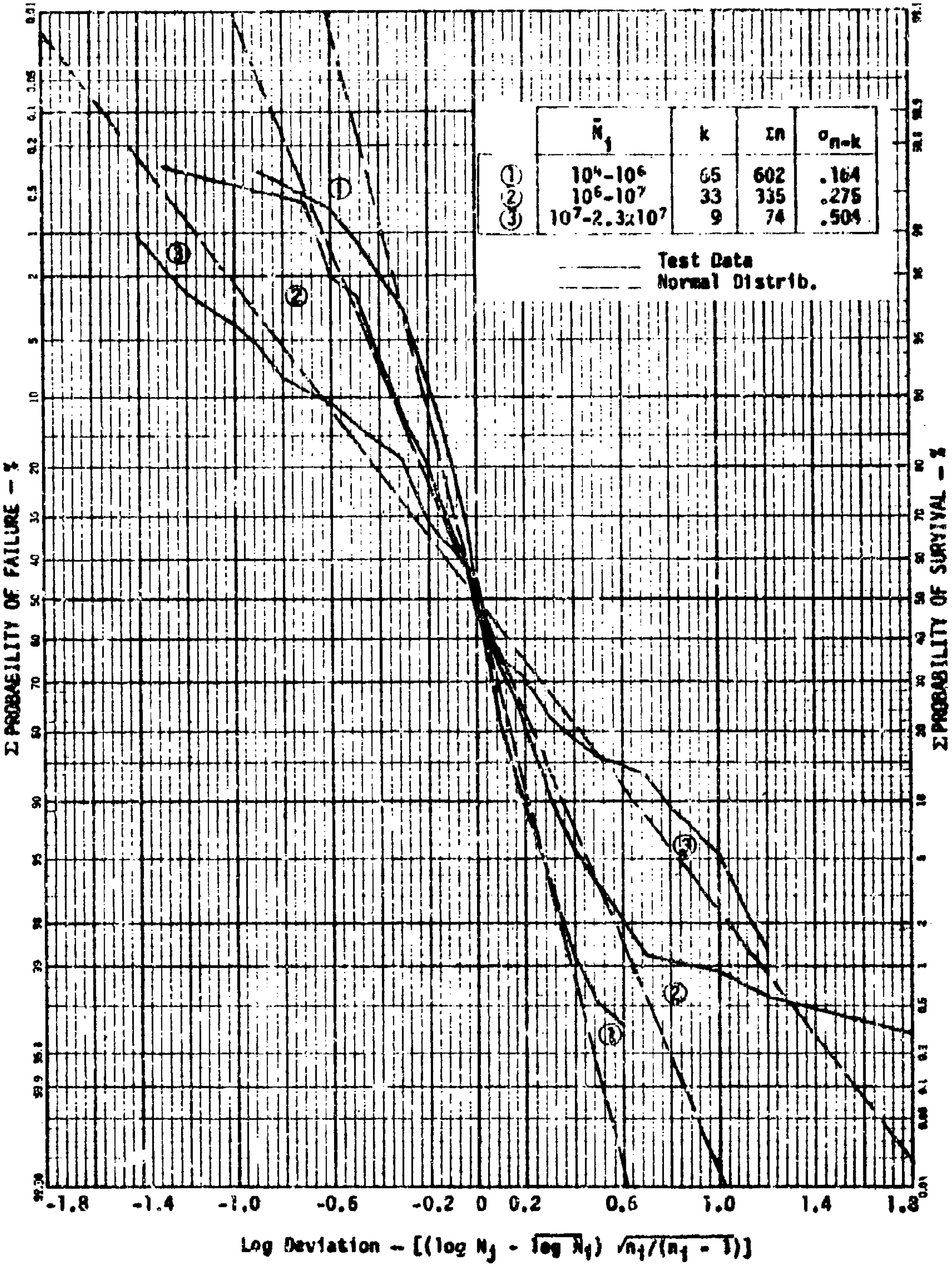


FIGURE 24 PROBABILITY DISTRIBUTIONS OF SPECTRUM LOADING UNNOTCHED SPECIMEN TEST LIVES.

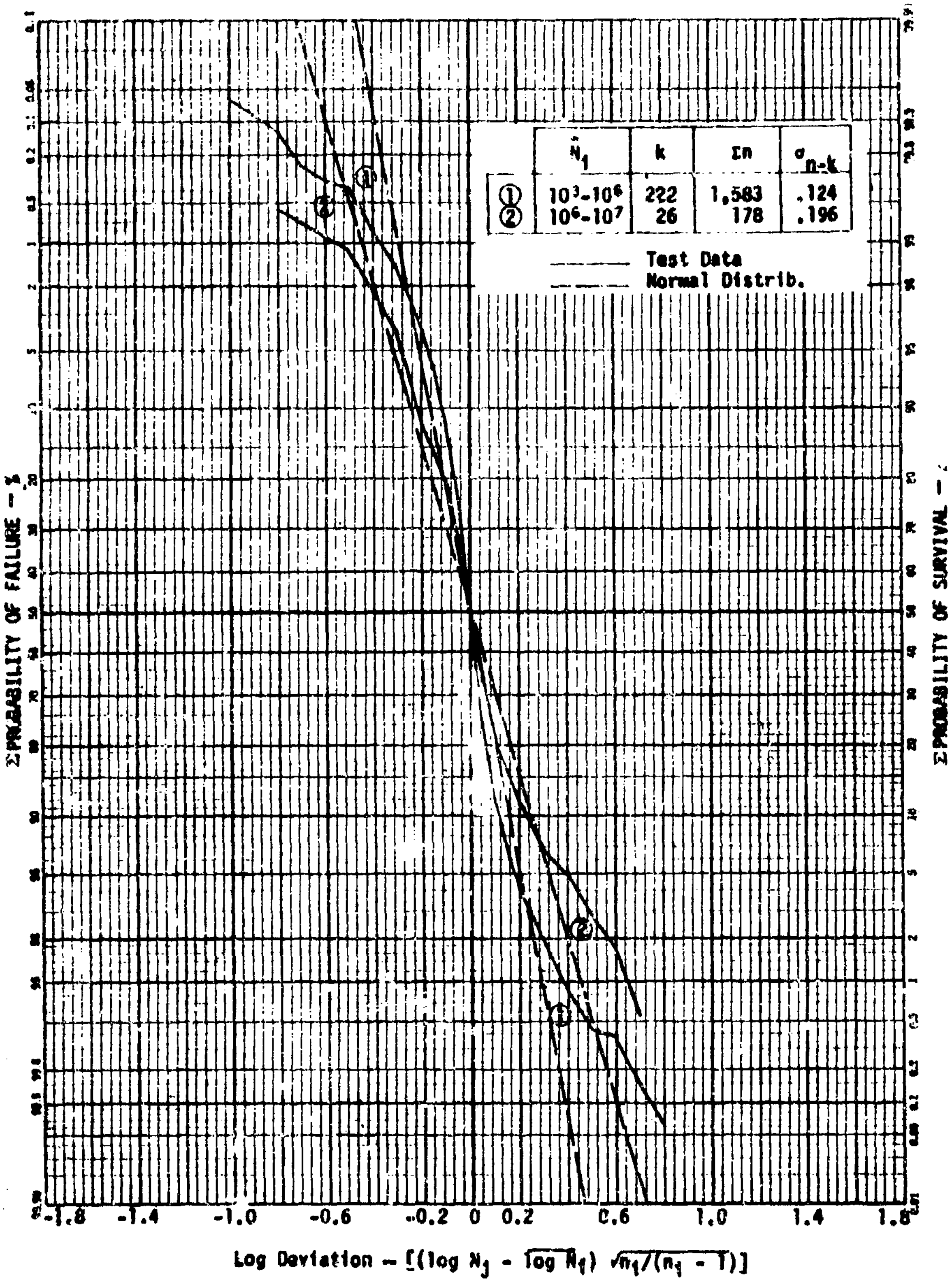


FIGURE 25. PROBABILITY DISTRIBUTIONS OF SPECTRUM LOADING NOTCHED SPECIMEN TEST LIVES.

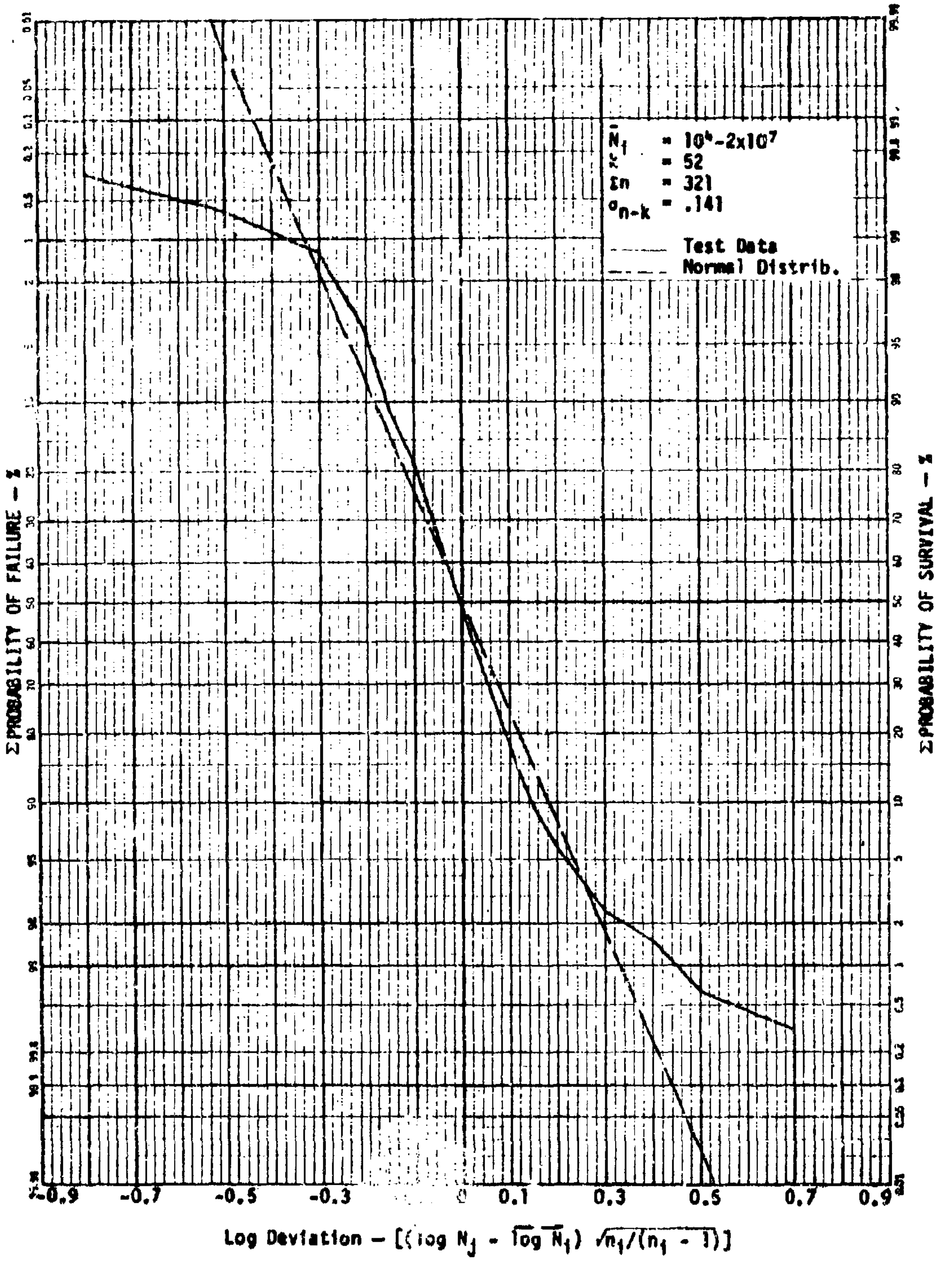


FIGURE 26. PROBABILITY DISTRIBUTIONS OF SPECTRUM LOADING STRUCTURAL COMPONENT SPECIMEN TEST LIVES.

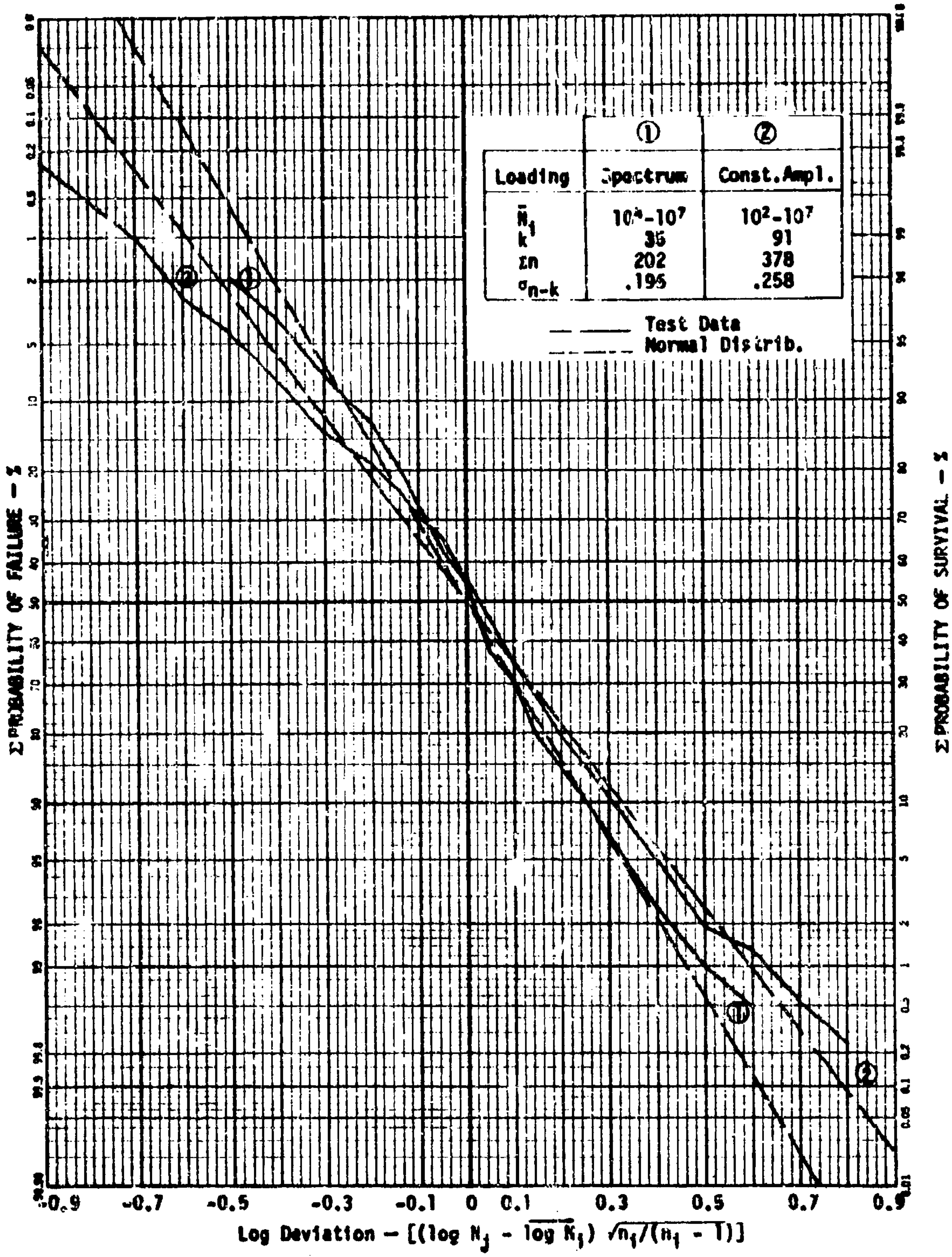


FIGURE 27. PROBABILITY DISTRIBUTIONS OF CONSTANT AMPLITUDE AND SPECTRUM LOADING FULL SCALE STRUCTURE TEST LIVES.

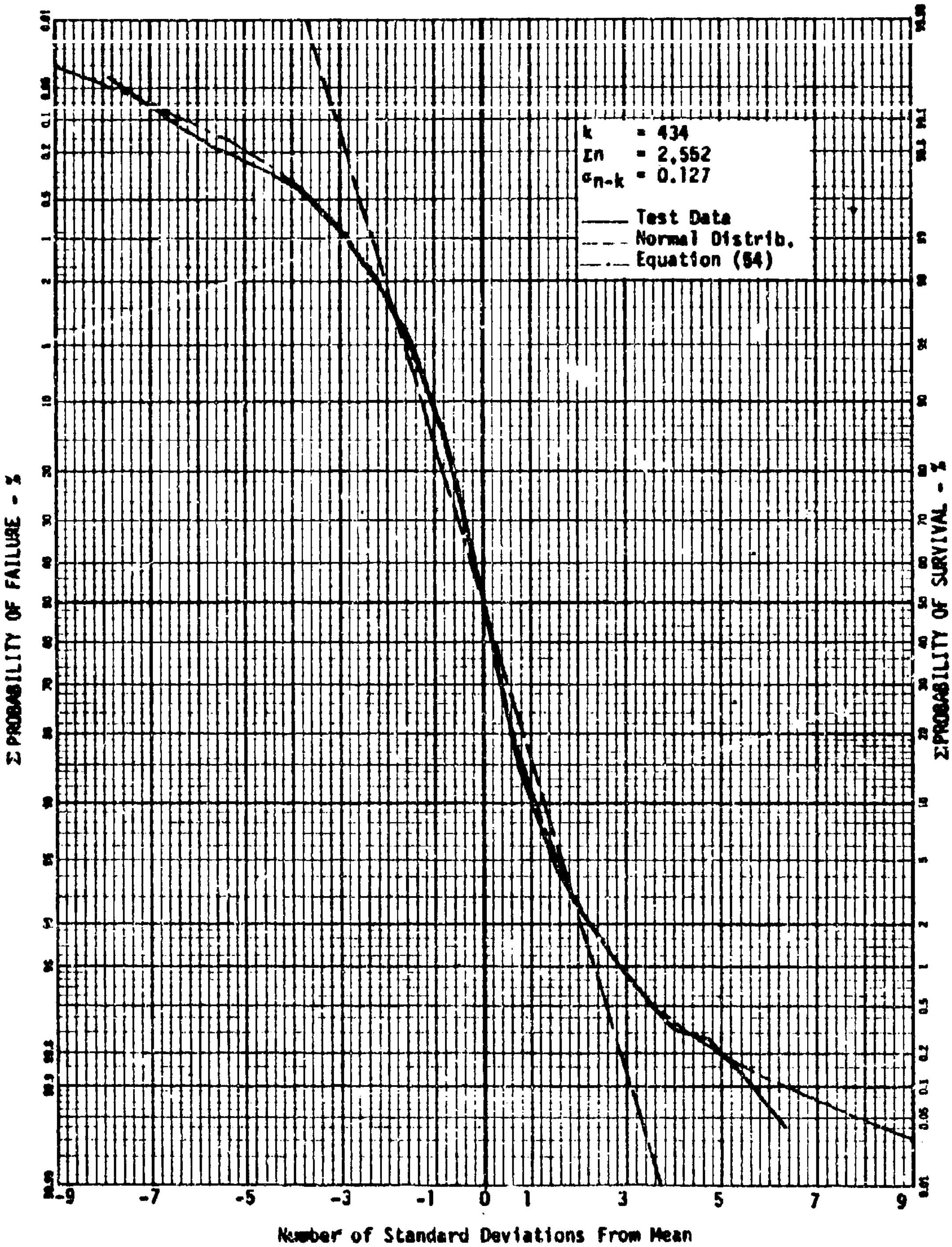


FIGURE 28. PROBABILITY DISTRIBUTION OF POOLED TEST DATA GROUPS WITH σ_{n-k}<0.15.

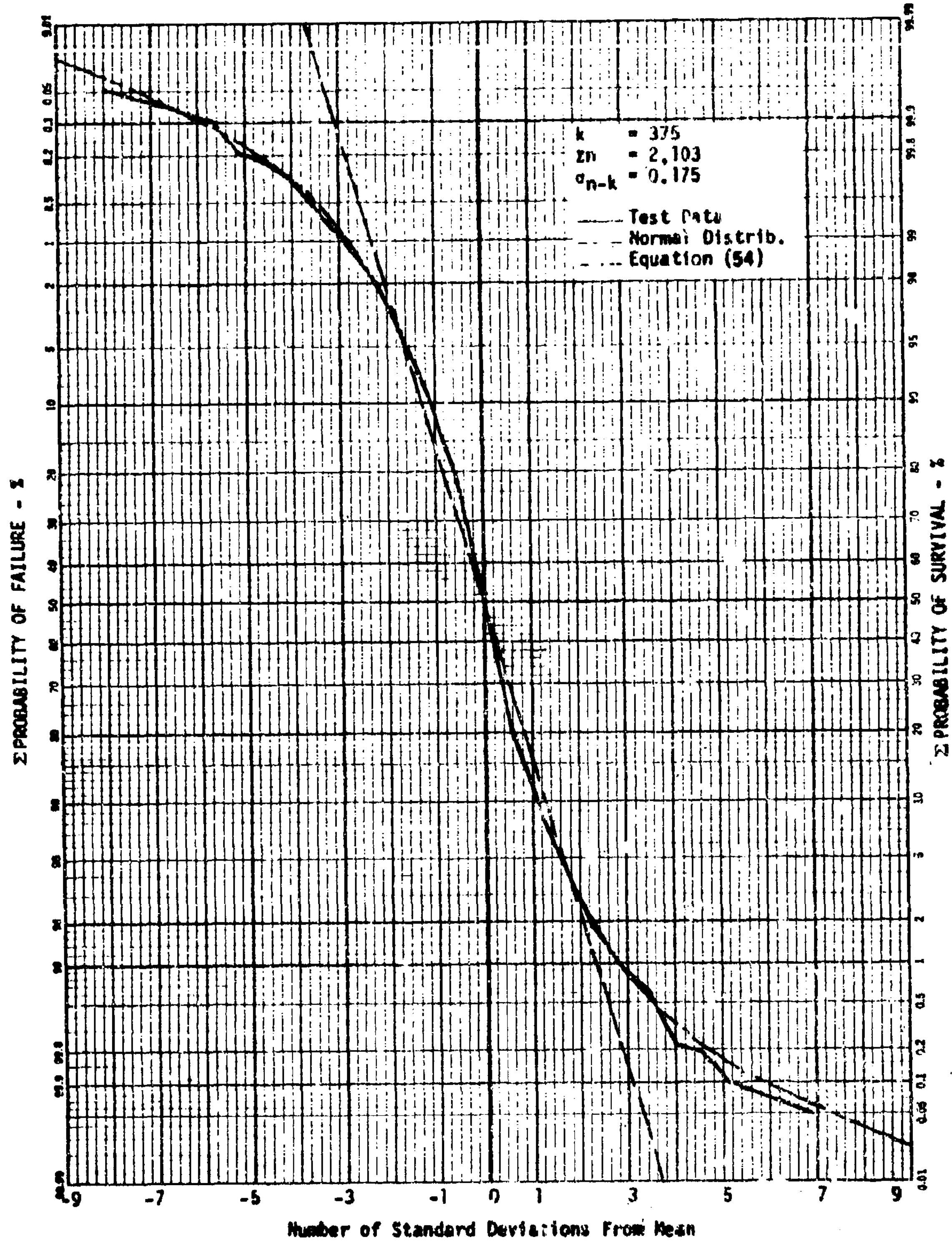


FIGURE 29. PROBABILITY DISTRIBUTION OF POOLED TEST DATA GROUPS WITH $0.15<\sigma_{n-k}<0.20$

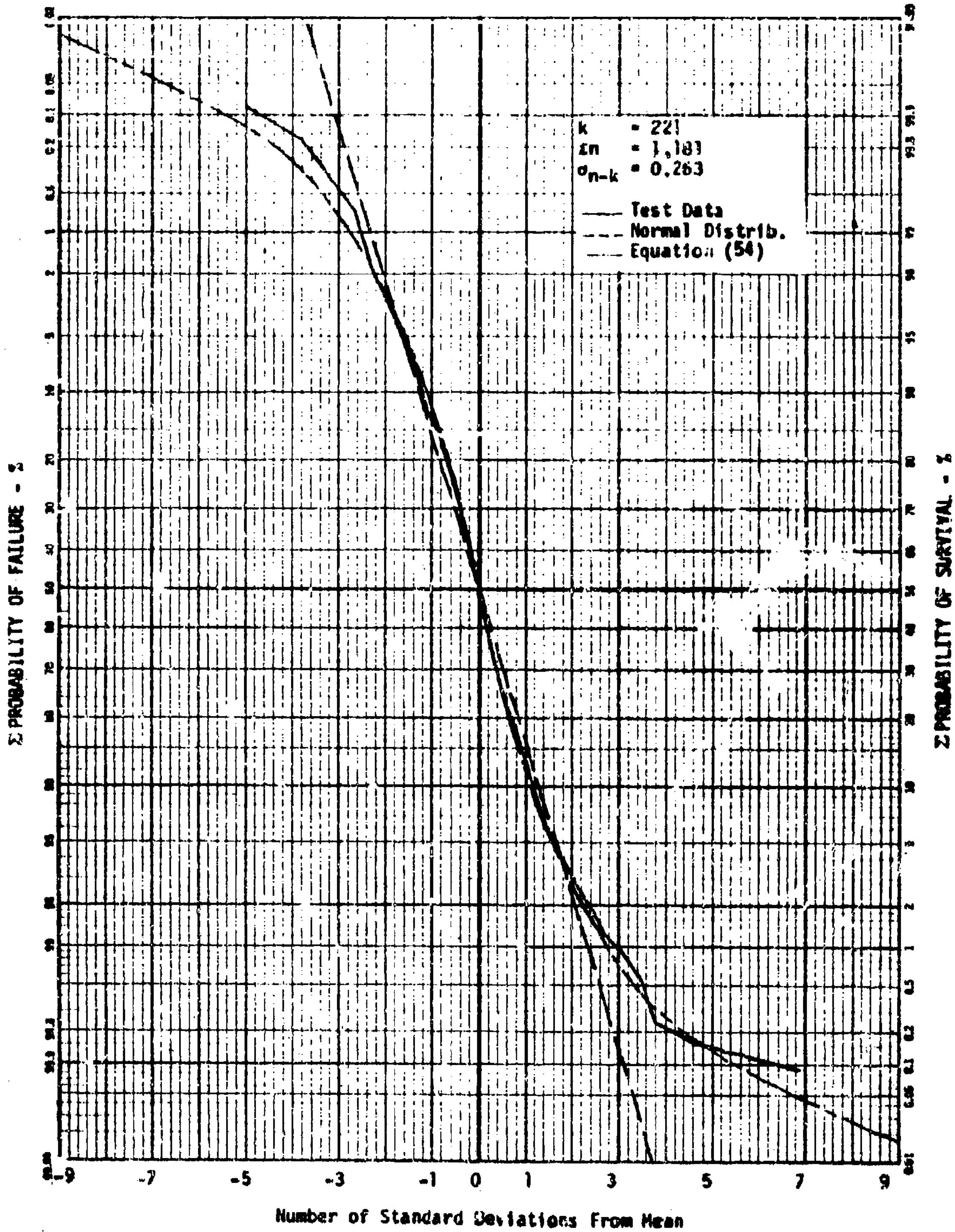


FIGURE 30. PROBABILITY DISTRIBUTION OF POOLED TEST DATA GROUPS WITH $0.20<\sigma_{n-k}<0.30$.

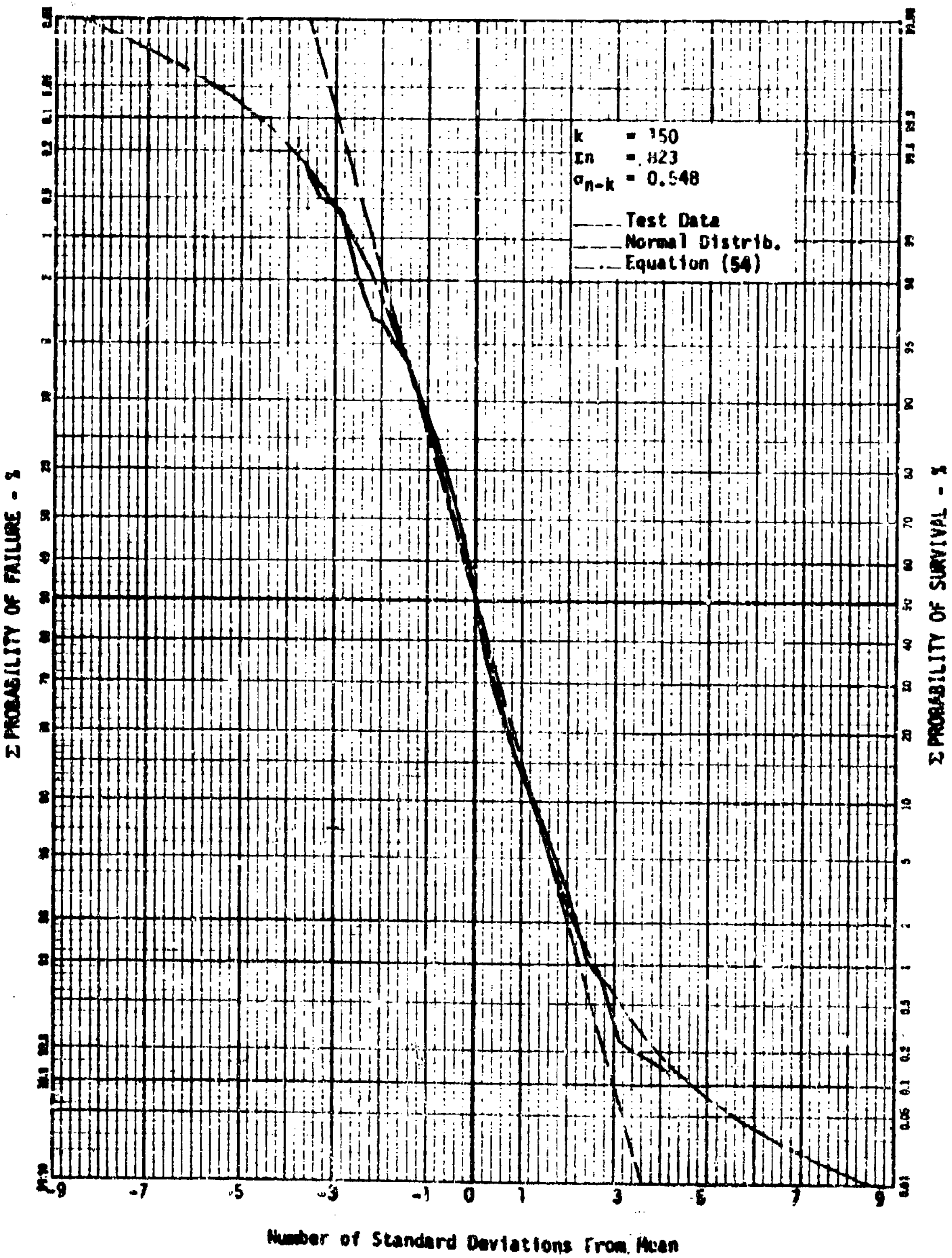


FIGURE 31. PROBABILITY DISTRIBUTION OF POOLED TEST DATA GROUPS WITH σ_{n-k}>0.30.

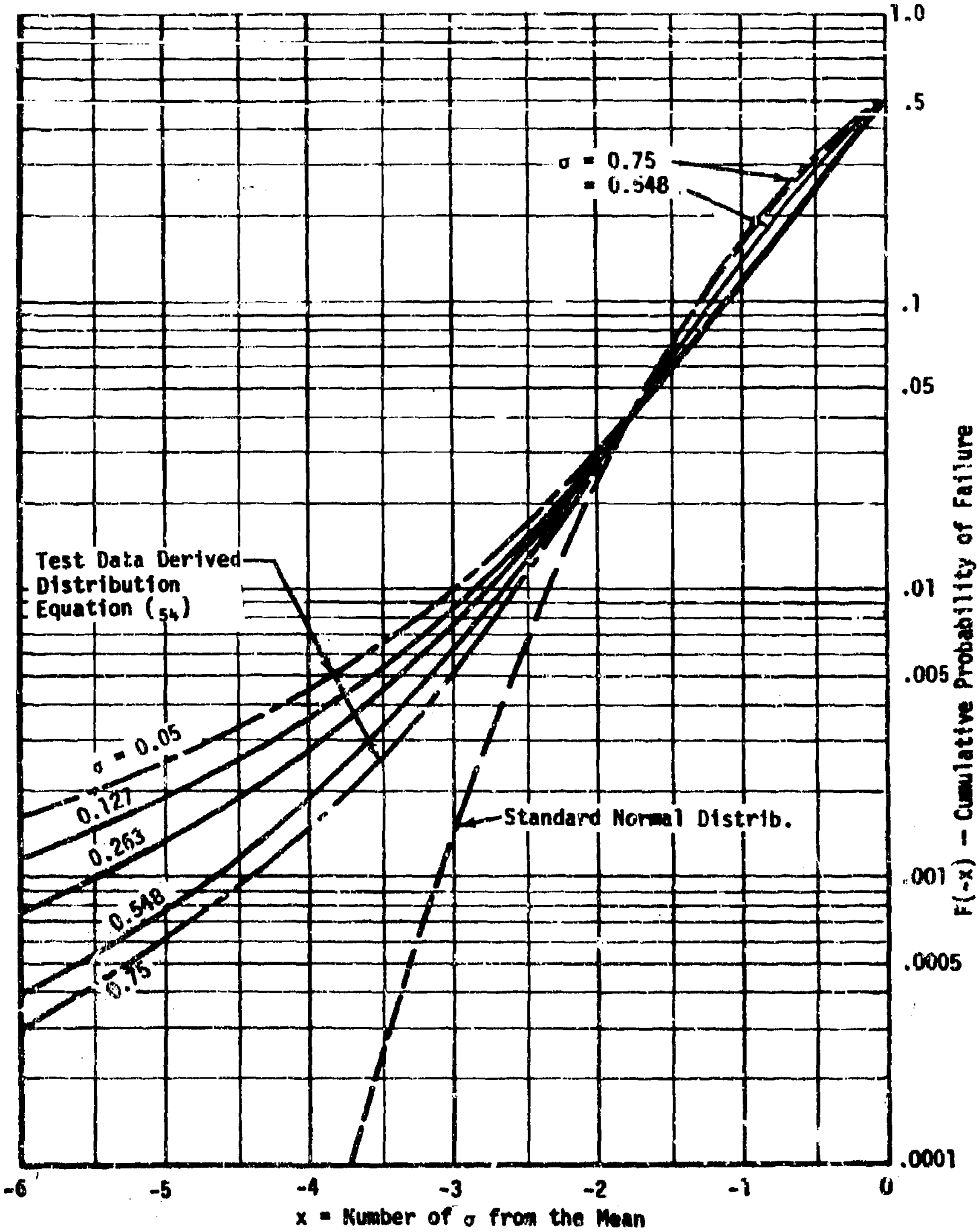


FIGURE 32. FATIGUE LIFE BASIC SCATTER PROBABILITY DISTRIBUTIONS

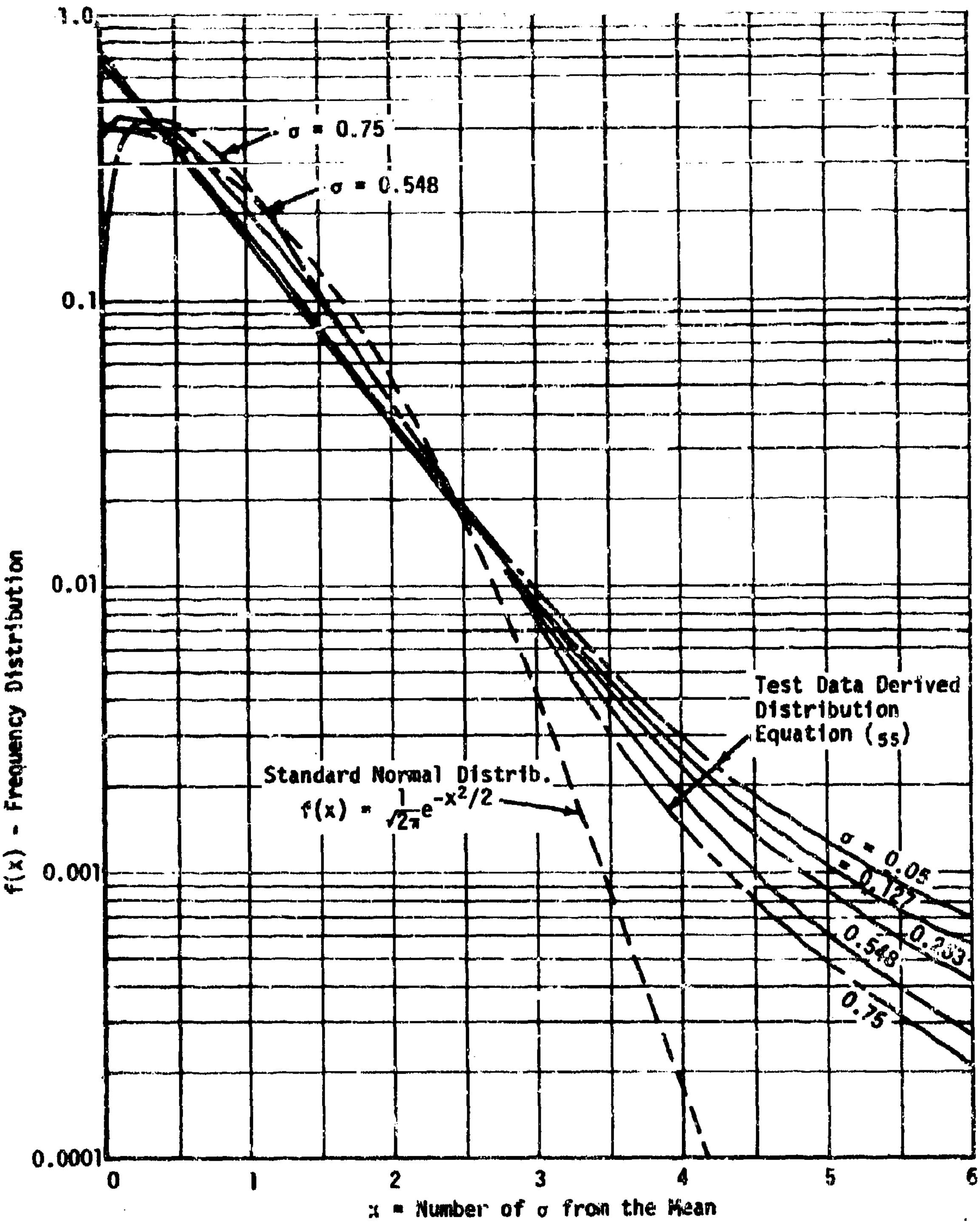


FIGURE 33. FATIGUE LIFE BASIC SCATTER FREQUENCY DISTRIBUTIONS

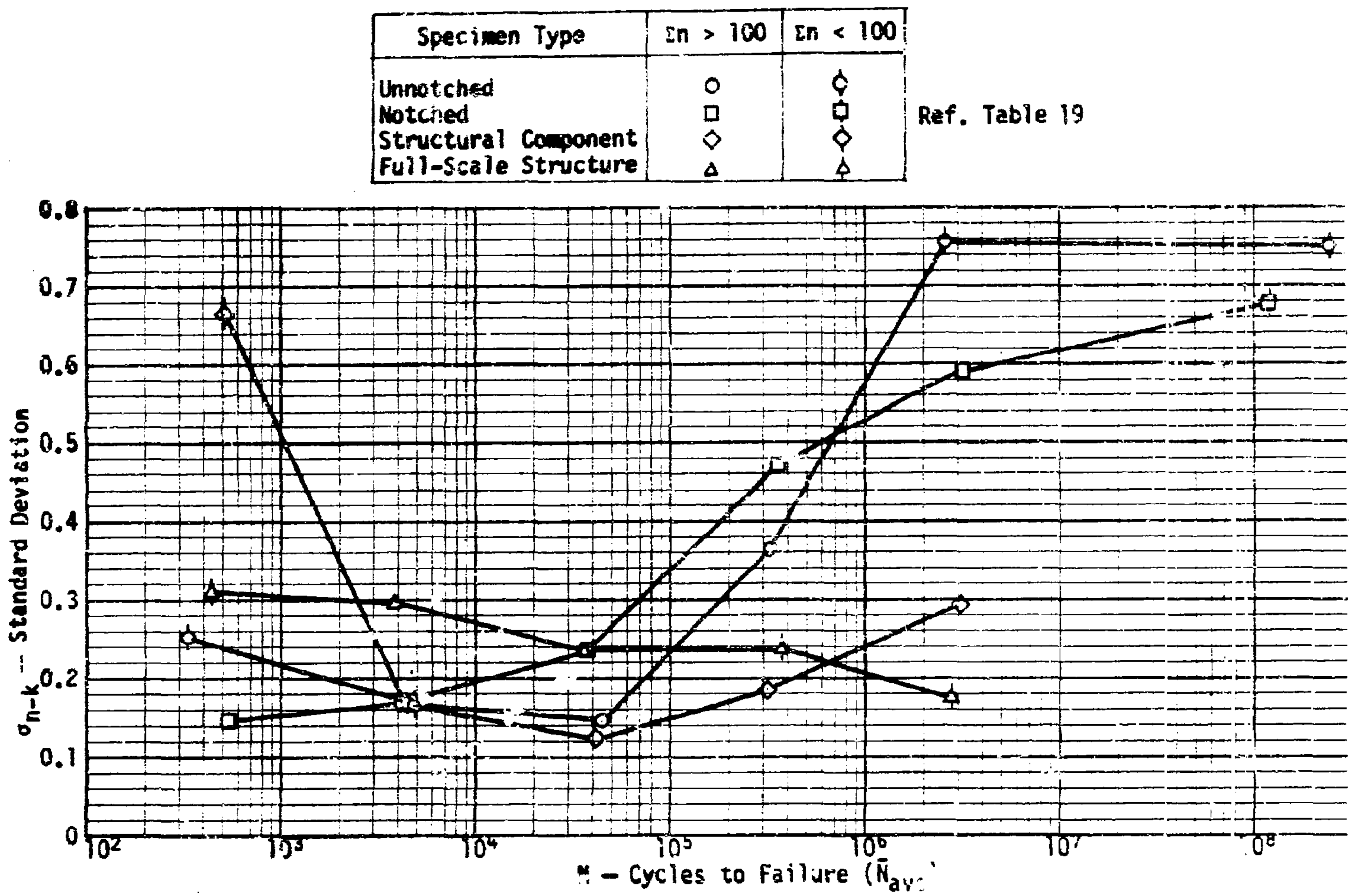


FIGURE 34. CONSTANT AMPLITUDE LOADING FATIGUE TEST LIFE SCATTER STANDARD DEVIATIONS.

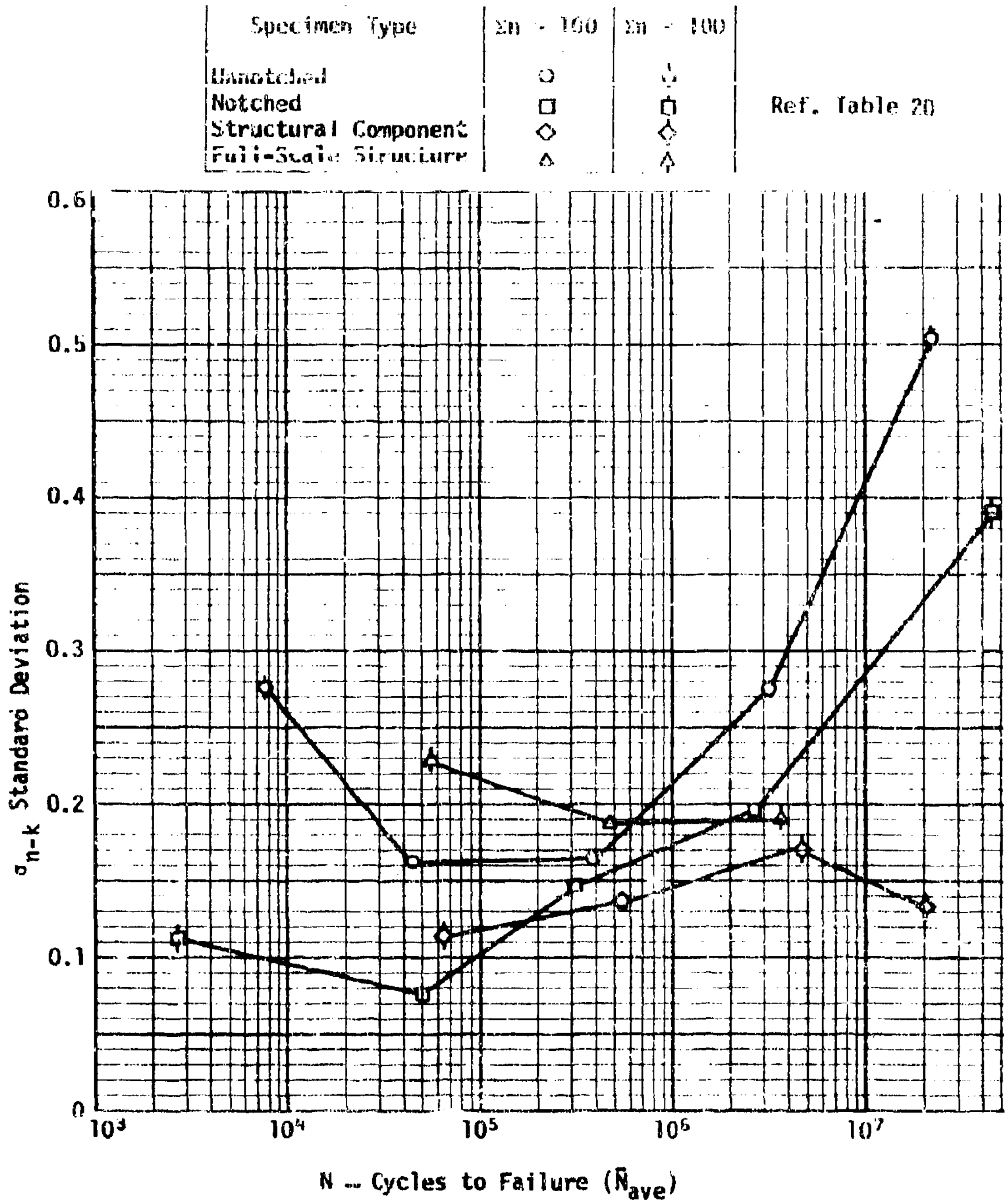


FIGURE 35. SPECTRUM LOADING FATIGUE TEST LIFE SCATTER STANDARD DEVIATIONS.

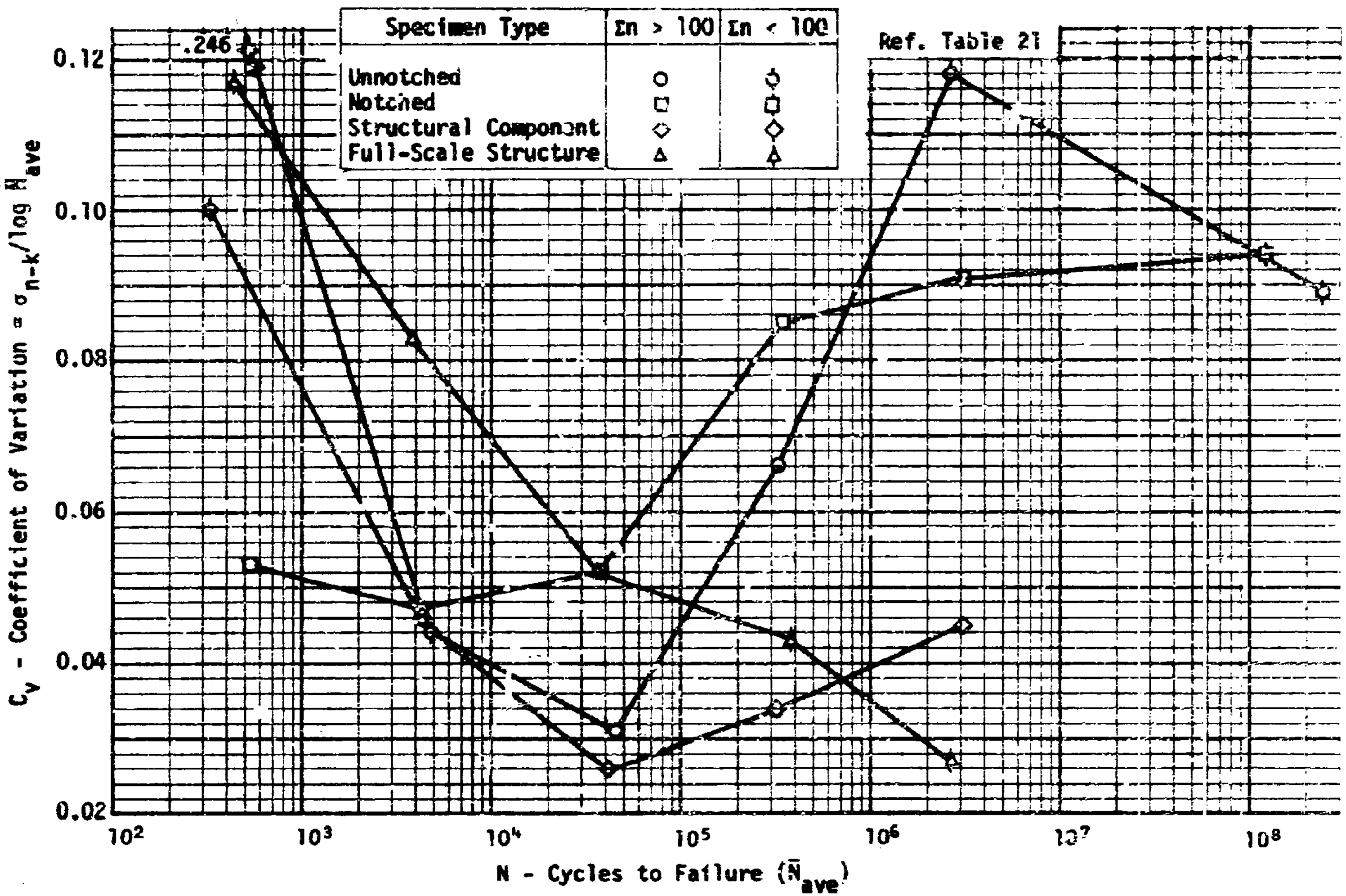


FIGURE 36. CONSTANT AMPLITUDE LOADING FATIGUE TEST LIFE SCATTER COEFFICIENT OF VARIATION.

Specimen Type	Σn > 100	Σn < 100
Unnotched	○	○
Notched	□	□
Structural Component	◇	◇
Full-Scale Structure	△	△

Ref. Table 21

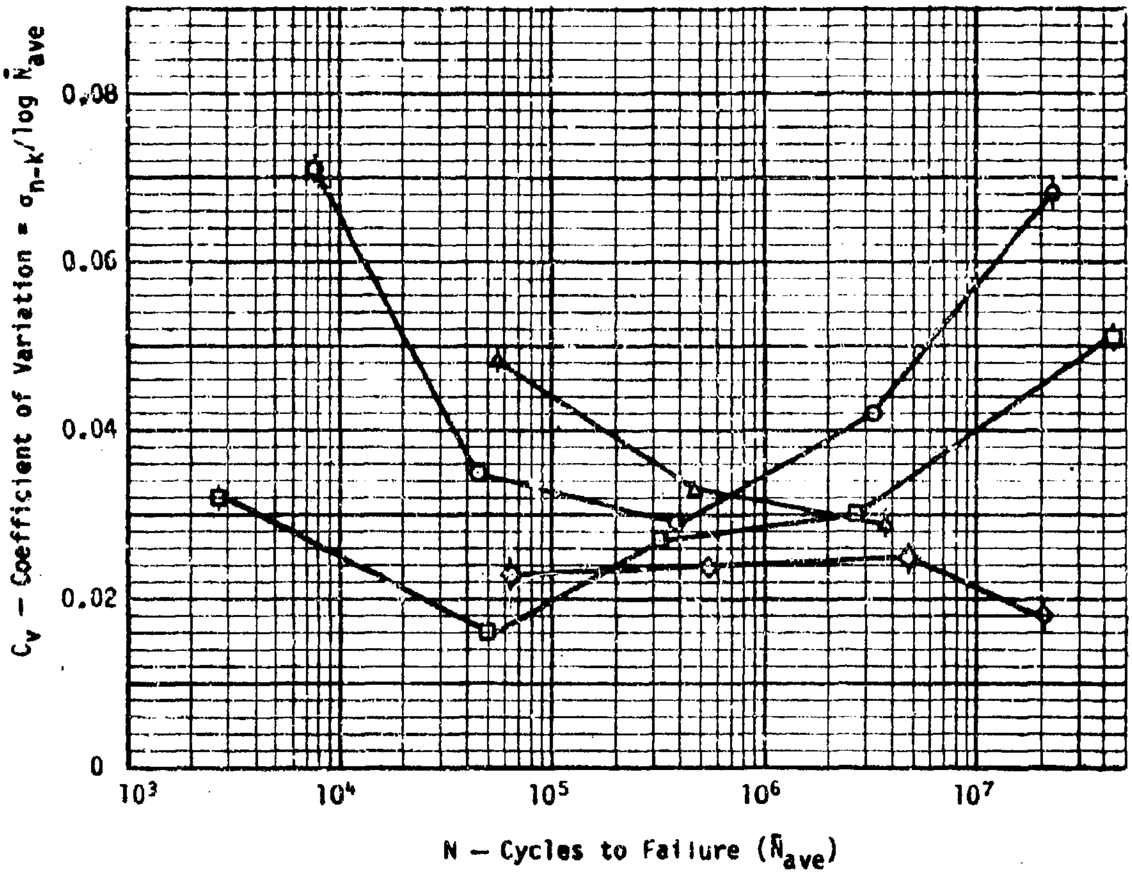


FIGURE 37. SPECTRUM LOADING FATIGUE TEST LIFE SCATTER COEFFICIENT OF VARIATION.

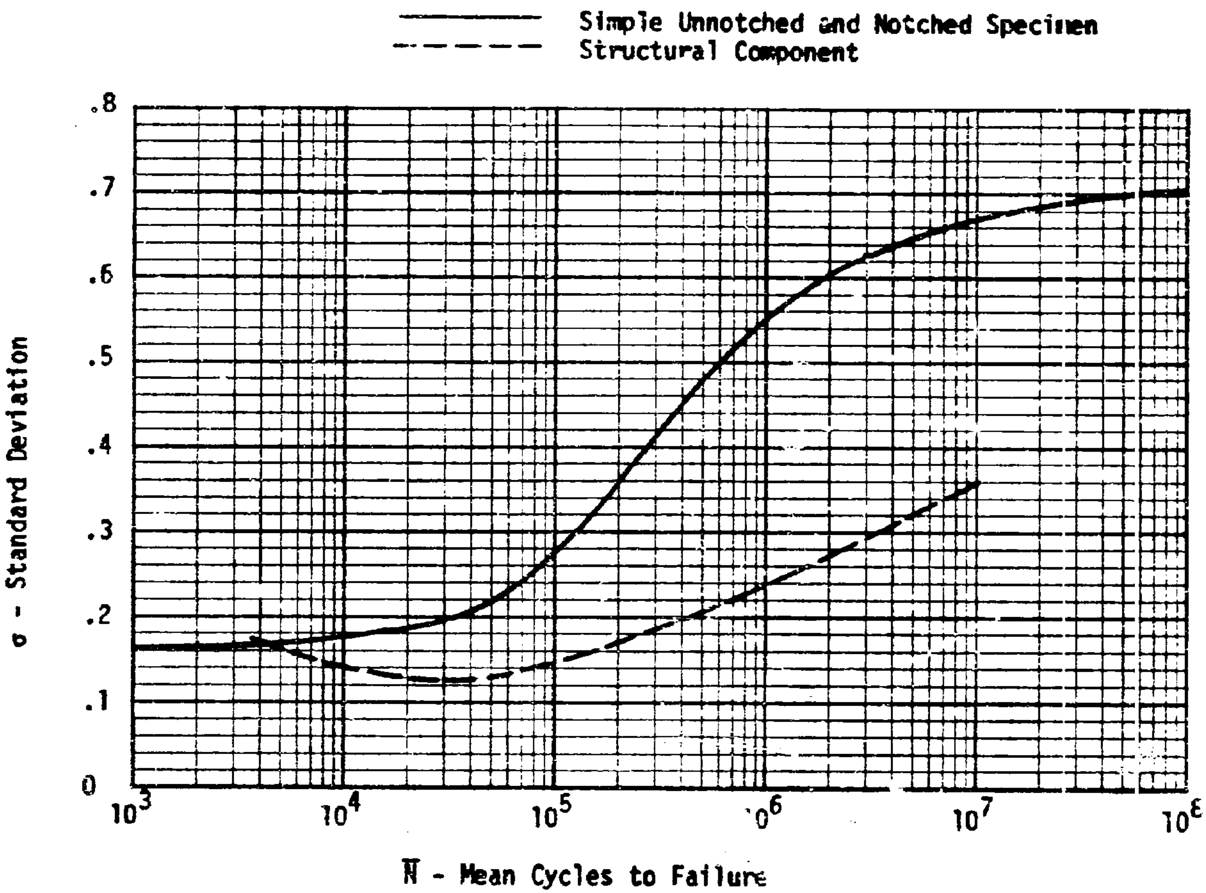


FIGURE 38. RECOMMENDED FATIGUE LIFE SCATTER STANDARD DEVIATIONS UNDER CONSTANT AMPLITUDE LOADING FOR ALUMINUM ALLOYS

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13. ABSTRACT Aircraft fatigue strength design and analysis concepts were investigated in the areas of fatigue life scatter factors and fatigue strength design-analysis charts. A fatigue scatter factor is defined as the ratio of the mean life to the life for a specified probability of failure and confidence level. For design purposes, operational life scatter factors are defined in terms of the joint probability distribution of the applied loads spectra variation in a fleet of aircraft and the basic fatigue life scatter represented by fatigue test data. Basic fatigue life scatter properties for aluminum alloy materials and structures were statistically derived from a fatigue test data survey of over 6,000 specimens. The basic scatter derived frequency and probability distributions greatly deviate from the log Normal distribution beyond $\mu \pm 2\sigma$. Several joint probability distribution models illustrate the procedure of calculating operational life scatter factors. An actual aircraft service failure history is accurately predicted by the joint probability distribution concept. A procedure for the development of fatigue strength design-analysis charts is outlined and illustrated by several examples. The charts, in the form of damage rate curves, are defined by generalized loads spectra parameters and the fatigue quality of the structural element.

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